S.E. Sem. III [CIVIL]

Applied Mathematics-III

Time : 3 Hrs.] Prelim Question Paper **[Marks : 80**

N.B.: (1) Question No. **1** is **compulsory**.
(2) Attempt any **THREE** of the remaining.
(3) Figures to the right indicate full marks.

1. (a) Find Laplace Transformation of $\dfrac{\sin(3t)}{t}$ **[5]**

(b) Prove that $f(z) = \cosh z$ is analytic and find it's derivative **[5]**

(c) Find Fourier series for $f(x) = 16 - x^2$ over $(-4, +4)$ **[5]**

(d) Evaluate using Cauchy's Residue Theorem $\int_C z^4 e^{1/z} dz$, where $C: |z| = 1$ **[6]**

2. (a) Expand $f(z) = \dfrac{1}{(z-1)(z-2)}$ in (i) $1 < |z - 1| < 2$ (ii) $|z| < 1$ **[6]**

(b) Find the Fourier series for $f(x) = \dfrac{x-\pi}{4}$; $0 \le x \le 2\pi$. **[6]**

Hence prove that $1 - \dfrac{1}{3} + \dfrac{1}{5} - \dfrac{1}{7} + \ldots = \dfrac{\pi}{4}$

(c) Find Inverse Laplace transform of **[8]**

(i) $\dfrac{s+19}{(s+9)(s^2+4)}$ (ii) $\dfrac{e^{-3s}}{(s^2+10s+29)}$

3. (a) Find the analytic function $f(z) = u + iv$ **[6]**

if $u + v = \dfrac{2\sin(2x)}{e^{2y} + e^{-2y} - 2\cos(2x)}$

(b) Evaluate $\int_C \dfrac{e^{2z}}{(z-\pi i)^3} dz$ where C is $|z - 2i| = 2$ **[6]**

(c) Solve the differential equation $\dfrac{d^2y}{dt^2} + 4y = f(t)$ with $y(0) = 0$ and $y'(0) = 1$ and $f(t) = \begin{matrix} 1 & 0 < t < 1 \\ 0 & 1 < t \end{matrix}$ **[8]**

4. (a) Find the orthogonal Trajectory of $3x^2 - 2x^2y + y^2 = \text{constant}$ **[6]**

(b) Determine the solution of one dimensional heat equation $\frac{\partial y}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ **[6]**
under the boundary conditions
$u(0, t) = 0$, $u(1, t) = 0$ and $u(x, 0) = x, 0 < x < .$

(c) Find the Fourier series representation of $f(x) = x^2[-1, 1]$ **[8]**
Hence find the sum : (i) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ (ii) $\sum_{n=1}^{\infty} \frac{1}{n^2}$

5. (a) Find Inverse Laplace Transform of $\frac{s}{s^4 + 8s^2 + 16}$ using Convolution theorem **[6]**

(b) Find the Bilinear Transform which transform the points z = 2, i, –2 of z-plane into the points w = 1, i, –1 of the w-plane respectively. Also find fixed points of this transformation. **[6]**

(c) Evaluate $\int_{-\infty}^{\infty} \frac{x^2}{x^6 + 1} dx$ **[8]**

6. (a) Evaluate $\int_C \left[x^2 - 2ixy\right] dz$ along $y = 2x^2$ From z = 0 to z = 3 + 18i **[6]**

(b) Obtain the complex form of Fourier series for the function $f(x) = e^{4x}$ in $0 < x < 4$ **[6]**

(c) Find half range sine series of the function $f(x) = x(3-x)$ in $0 \le x \le 3$ **[8]**
Hence prove that

(i) $\frac{1}{1^6} + \frac{1}{3^6} + \frac{1}{5^6} + \frac{1}{7^6} + ... = \frac{\pi^6}{960}$ (ii) $\sum_{n=1}^{\infty} \frac{1}{(n)^6} = \frac{\pi^6}{945}$

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