

S.E. Sem. III [CMPN]  
**Discrete Structures**  
Prelim Question Paper

Time : 3 Hrs.]

[Marks : 80

**N.B.:** (1) Question No. 1 is compulsory.

(2) Attempt any three questions from remaining five questions.

(3) All questions carry equal marks as indicated by figures to the right.

(4) Assumptions made should be clearly stated.

1. (a) How many integers between 1 and 60 are not divisible by 2 nor by 3 not by 5? [6]

(b) Use induction to show that,  $1 + a + a^2 + \dots + a^{n-1} = \frac{a^n - 1}{a - 1}$ ,  $a \neq 1$ . [6]

(c) Let  $A = \{a, b, c, d, e\}$  and  $R = \{(a, a), (a, b), (b, c), (c, e), (c, d), (d, e)\}$ . Compute  $R^2$  and  $R^\infty$ . [8]

2. (a) Show that in a group,  $\forall a, b \in G, (a * b)^2 = a^2 * b^2$ , iff  $(G, *)$  must be abelian. [6]

(b)  $f : \mathbb{R} - \left\{ \frac{2}{5} \right\} \rightarrow \mathbb{R} - \left\{ \frac{4}{5} \right\}$  defined by  $f(x) = \frac{4x + 3}{5x - 2}$  [6]

show that the function is bijective and find rule for  $f^{-1}$ .

(c) Let  $H = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$  [8]

be a parity check matrix. Decode the following word related to maximum likelihood technique (Decoding function) associated with  $e_H$ .

Decode the following :

(i) 10100

(ii) 01101

(iii) 11011

3. (a) Show that if 30 dictionaries in a library contain a total of 61,327 pages, then one of the dictionaries must have at least 2045 pages. [6]

(b) Show that the set of all divisors of 70 forms a lattice. [6]

(c)  $f : \mathbb{R} \rightarrow \mathbb{R}$  is defined by  $f(x) = x^3$  [8]

$g : \mathbb{R} \rightarrow \mathbb{R}$  is defined by  $g(x) = 4x^2 + 1$

$h : \mathbb{R} \rightarrow \mathbb{R}$  is defined by  $h(x) = 7x - 2$

Find the rule defining

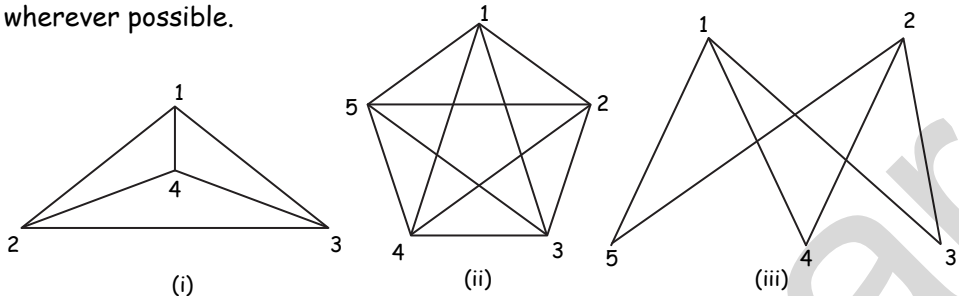
(i)  $f \circ g$

(ii)  $g \circ f$

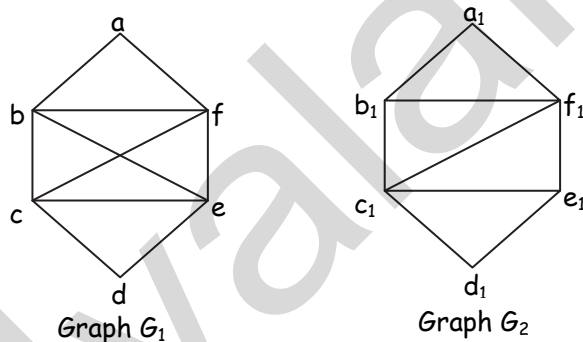
(iii)  $(g \circ h) \circ f$

(iv)  $g \circ (h \circ f)$

4. (a) Decide which of the following graphs are Eulerian or Hamiltonian or both and write down as Eulerian circuit and Hamiltonian circuit wherever possible. [6]



- (b) Let  $G = \{0, 1, 2, 3, 4, 5\}$  [6]
- Prepare composition table with respect to '+<sub>6</sub>'
  - Prove that  $G$  is an abelian group with respect to '+<sub>6</sub>'
  - Find the inverse of 2, 3 and 5.
  - Is it cyclic?
  - Find the order of 2, 3 and sub groups generated by these elements.
- (c) Determine whether following graphs are isomorphic : [8]



5. (a) Use the laws of logic to show that  $[(p \rightarrow q) \wedge \sim q] \rightarrow \sim p$  is a tautology. [6]
- (b) Let  $A = \{1, 2, 3, 4\}$  and  $R = \{(1, 2), (2, 3), (3, 4), (2,1)\}$ . Find the transitive closure using Warshalls Algorithm. [6]
- (c) Consider  $(2, 6)$  encoding function  $e : B^2 \rightarrow B^6$  defined as [8]
- $e(00) = 000000$     $e(01) = 011110$     $e(10) = 101010$     $e(11) = 111000$
- Find the minimum distance
  - How many error can 'e' detect?
6. (a) Find formula for sequences with following first five terms. [6]
- $1, 1/2, 1/4, 1/8, 1/16$
  - $1, 3, 5, 7, 9$
  - $1, -1, 1, -1, 1$
- (b) A connected planar graph has 9 vertices having degrees 2, 2, 2, 3, 3, 3, 4, 4 and 5. How many edges are there? [6]
- (c) Determine the sequence whose recurrence relation is given by [8]
- $C_n = 3C_{n-1} - 2C_{n-2}$  with initial conditions  $C_1 = 5, C_2 = 3$ .

