S.E. Sem. III [CMPN]

# Discrete Structures

**Time : 3 Hrs.]** Prelim Question Paper Solution **[Marks : 80**

**Q.1(a) How many integers between 1 and 60 are not divisible by 2 nor by 3 not by 5?** **[6]**

**(A)** Let A be the set of integer divisible by 2

B be the set of integer divisible by 3

C be the set of integer divisible by 5

$$U = 60, |A| = \frac{60}{2} = 30, \ |B| = \frac{60}{3} = 20, \ |C| = \frac{60}{5} = 2,$$

$$|A \cap B| = \frac{60}{2 \times 3} = 10, \ |A \cap C| = \frac{60}{2 \times 5} = 6, \ |B \cap C| = \frac{60}{3 \times 5} = 4$$

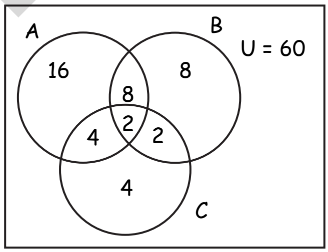
$$|A \cap B \cap C| = \frac{60}{2 \times 3 \times 5} = 2$$

Using principle of Inclusion and Exclusion

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$$

$x = 30 + 20 + 12 - 10 - 4 - 6 + 2$

$x = 44$




$\therefore$ $|A \cup B \cup C| = 44$

$\therefore$ 44 integers are either div by 2 or by 3 or by 5

$\therefore$ 16 integers are divisible only by 2

$\therefore$ 8 integers are divisible only by 3

$\therefore$ 4 integers are divisible only by 5

**Q.1(b) Use induction to show that, $1 + a + a^2 + \ldots + a^{n-1} = \frac{a^n - 1}{a - 1}$, $a \neq 1$.** **[6]**

**(A)** For n = 1,

L.H.S. = 1 R.H.S. = $\frac{a^1 - 1}{a - 1} = 1$

$\therefore$ the result is true for n = 1

Let the result is true for n = k

$$1 + a + a^2 + \ldots\ldots. + a^{k-1} = \frac{a^k - 1}{a - 1} \quad \text{[i.e. P(k)]}$$

Now we have to prove that P(k) $\Rightarrow$ P(k + 1)

i.e. $1 + a + a^2 + \ldots\ldots + a^k = \frac{a^{k+1} - 1}{a - 1}$

L.H.S. $= 1 + a + a^2 + \ldots.. + a^{k-1} + a^k$

$= \frac{a^k - 1}{a - 1} + a^k$ [by P(k)]

$= \dfrac{a^k - 1 + a^{k+1} + a^k}{a-1} \quad = \dfrac{a^{k+1} - 1}{a-1}$

= R.H.S.

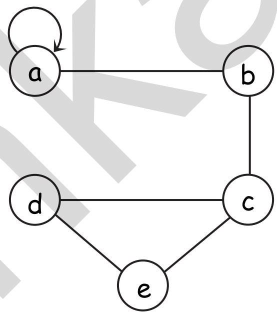
$\therefore$ P(k) $\Rightarrow$ P(k + 1)

Hence the result is true for n = k + 1

$\therefore$ The result is true for all n.

**Q.1(c) Let A = {a, b, c, d, e} and R = {(a, a), (a, b), (b, c), (c, e), (c, d), (d, e)}. Compute $R^2$ and $R^\infty$.** **[8]**

**(A)** The digraph of R is shown in Figure,




x $R^2$ y means that there is a path of length n from x to y in R.

| | | | | |
|---|---|---|---|---|
| a $R^2$ a | Since | a R a | and | a R a |
| a $R^2$ b | Since | a R a | and | a R b |
| a $R^2$ c | Since | a R b | and | b R c |
| b $R^2$ e | Since | b R c | and | c R e |
| b $R^2$ d | Since | b R c | and | c R d |
| c $R^2$ e | Since | c R d | and | d R e |

Hence $R^2$ = {(a, a), (a, b), (a, c), (b, e), (b, d), (c, e)}

To compute $R^\infty$, we need all ordered pairs of vertices for which there is a path of any length from the first vertex to the second. From Figure, we see that,

$R^\infty$ = {(a, a), (a, b), (a, c), (a, d), (a, e), (b, c), (b, d), (b, e), (c, d), (c, e), (d, e)}

For example,

(a, d) $\in R^\infty$, since there is a path of length 3 from a to d : a, b, c, d

Similarly, (a, e) $\in R^\infty$, since there is a path of length 3 from a to e : a, b, c, e

**Q.2(a) Show that in a group, $\forall$ a, b $\in$ G, $(a * b)^2 = a^2 * b^2$, iff (G, *) must be abelian.** **[6]**

**(A)** If part : $\forall$ a, b $\in$ G, $(a * b)^2 = a^2 * b^2$ (given)

$\Rightarrow$ $(a * b) * (a * b) = (a * a) * (b * b)$

$\Rightarrow$ $a * (b * a) * b = a * (a * b) * b$ (by associativity)

$\Rightarrow$ $b * a = a * b$ by left & right cancellation law.

$\Rightarrow$ $*$ is commutative.

$\Rightarrow$ (G, *) is abelian group.

**Only if part :**

Let (G, *) be an abelian group.

$\therefore$ $a * b = b * a$

We have to prove that

$$(a * b)^2 = a^2 * b^2$$

$$\begin{aligned}
\text{LHS} &= (a * b)^2 \\
&= (a * b) * (a * b) \\
&= a * (b * a) * b && \dots \text{ by associativity} \\
&= a * (a * b) * b && \dots \text{ by commutativity} \\
&= (a * a) * (b * b) && \dots \text{ by associativity} \\
&= a^2 * b^2
\end{aligned}$$

**Q.2(b)** $f : R - \left\{\frac{2}{5}\right\} \to R - \left\{\frac{4}{5}\right\}$ **defined by** $f(x) = \frac{4x+3}{5x-2}$ **[6]**

**show that the function is bijective and find rule for $f^{-1}$.**

**(A)** $\because$ $f : R - \left\{\frac{2}{5}\right\} \to R - \left\{\frac{4}{5}\right\}$ is defined by $f(x) = \frac{4x+3}{5x-2}$

(i) Injective :

Consider $f(x_1) = f(x_2)$

$$\frac{4x_1+3}{5x_1-2} = \frac{4x_2+3}{5x_2-2}$$

or $(4x_1 + 3)(5x_2 - 2) = (4x_2 + 3)(5x_1 - 2)$

or $20x_1x_2 - 8x_1 + 15x_2 - 6 = 20x_1x_2 - 8x_2 + 15x_1 - 6$

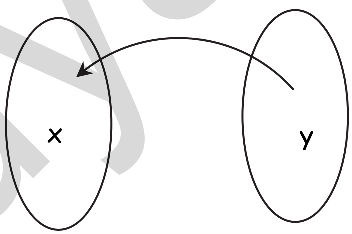
or $-8x_1 - 15x_1 = -8x_2 - 15x_2$

or $-23x_1 = -23x_2$

or $x_1 = x_2$

$\therefore$ f is injective.

(ii) Surjective :




R – {2/5} (domain) R – {4/5} (codomain)

Consider an arbitrary element y in R – {4/5} (codomain)

Let $y = f(x)$

$$y = \frac{4x+3}{5x-2}$$

or $5xy - 4x = 4x + 3$

or $5xy - 4x = 2y + 3$

or $x(5y - 4) = 2y + 3$

or $x = \dfrac{2y+3}{5y-4}$

$\Rightarrow$ $\forall\ y \in R - \left\{\dfrac{4}{5}\right\}$ (codomain)

$\exists$ pre image $x \in R - \left\{\dfrac{2}{5}\right\}$ (domain)

$\Rightarrow$ Range of f = codomain

$\Rightarrow$ f is surjective

$\because$ f is injective and surjective both

$\therefore$ f is bijective

$\therefore$ $f^{-1}$ exist

$y = f(x) \Rightarrow x = f^{-1}(y)$

$\Rightarrow$ $x = \dfrac{2y+3}{5y-4} = f^{-1}(y)$

$\therefore$ The rule for $f^{-1}$ is

$f^{-1}(x) = \dfrac{2x+3}{5x-4}$

**Q.2(c) Let** $H = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ **[8]**

**be a parity check matrix. Decode the following word related to maximum likelihood technique (Decoding function) associated with $e_H$. Decode the following :**

**(i) 10100 (ii) 01101 (iii) 11011**

**(A)** Here, m = 2, n = 5

$\therefore$ We have $B^2$ = {00, 01, 10, 11}

$e(00) = 00\ x_1\ x_2\ x_3$

$[\ x_1\ x_2\ x_3\ ] = [0\ \ 0]\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} = [\ 0\ 0\ 0\ ]$

$\Rightarrow$ $x_1 = x_2 = x_3 = 0$

$e(01) = 01\ x_1\ x_2\ x_3$

$[\ x_1\ x_2\ x_3\ ] = [0\ \ 1]\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} = [\ 1\ 0\ 1\ ]$

$\Rightarrow$ $x_1 = 1, x_2 = 0, x_3 = 1$

$$e(10) = 10\,x_1\,x_2\,x_3$$

$$[\,x_1\,x_2\,x_3\,] = [1\;\;0]\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} = [\,0\;1\;1\,]$$

$\Rightarrow$ $x_1 = 0, x_2 = 1, x_3 = 1$

$$e(11) = 11\,x_1\,x_2\,x_3$$

$$[\,x_1\,x_2\,x_3\,] = [1\;\;1]\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} = [\,1\;1\;0\,]$$

$\Rightarrow$ $x_1 = 1, x_2 = 1, x_3 = 0$

Hence, $e_H : B^2 \to B^5$ is defined as

$e(00) = 00000 = x_0$ $\qquad$ $e(01) = 01101 = x_1$

$e(10) = 10011 = x_2$ $\qquad$ $e(11) = 11110 = x_3$

(i) let $x_t = 10100$

$|x_0 \oplus x_1| = |x_1| = 2$

$|x_1 \oplus x_1| = |11001| = 3$

$|x_2 \oplus x_1| = |00111| = 3$

$|x_3 \oplus x_1| = |01010| = 2$

$\Rightarrow$ Minimum distance is not unique.

**Note :** If minimum distance for $x_t$ is not unique, then we see on priority basis which one comes first.

The required nearer word to $x_t$

$d(x_t) = d(x_0) = 00$

$\therefore$ decode word for 10100 is 00

(ii) $x_t = 01101$

$|x_0 \oplus x_1| = |x_1| = 3$

$|x_1 \oplus x_1| = |00000| = 0$

$|x_2 \oplus x_1| = |11110| = 4$

$|x_3 \oplus x_1| = |10011| = 3$

$d(x_t) = d(x_1) = 01$

$\therefore$ decode word for 01101 is 01.

(iii) $11011 = x_t$

$|x_0 \oplus x_1| = |x_1| = 4$

$|x_1 \oplus x_1| = |10110| = 3$

$|x_2 \oplus x_1| = |01000| = 1$

$|x_3 \oplus x_1| = |00101| = 2$

$d(x_t) = d(x_2) = 10$

$\therefore$ decode word for 11011 is 10.

**Q.3(a) Show that if 30 dictionaries in a library contain a total of 61,327 pages, then one of the dictionaries must have at least 2045 pages.** **[6]**

**(A)** There are 30 dictionaries (30 pigeonholes) and these contain 61,327 pages (61,327 pigeons).

$\because$ 30 < < 61,327

$\therefore$ Extended pigeon hole principle is applicable.

By extended pigeon hole principle, one pigeon hole must have atleast $\left\lfloor \frac{n-1}{m} \right\rfloor + 1$ pigeons.

Here, n = 61327, m = 30

$\therefore \quad \left\lfloor \frac{n-1}{m} \right\rfloor + 1 = 2044 + 1$

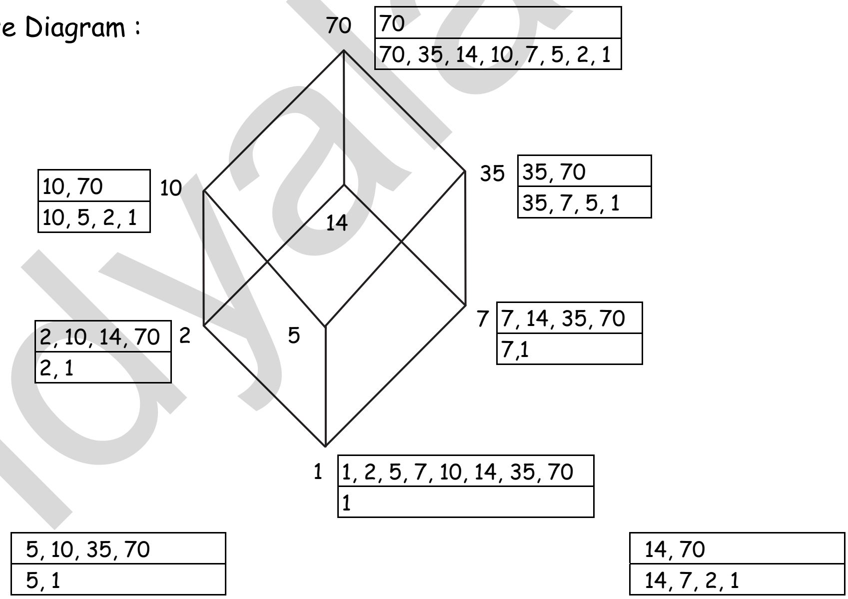
= 2045 pigeons.

$\therefore$ One dictionary must have atleast 2045 pages.

**Q.3(b) Show that the set of all divisors of 70 forms a lattice.** **[6]**

**(A)** Now, $D_{70}$ = {1, 2, 5, 7, 10, 14, 35, 70}

Hasse Diagram :



Now , for Least Upper Bound (LUB) :

| | 1 | 2 | 5 | 7 | 10 | 14 | 35 | 70 |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 5 | 7 | 10 | 14 | 35 | 70 |
| 2 | 2 | 2 | 10 | 14 | 10 | 14 | 70 | 70 |
| 5 | 5 | 10 | 5 | 35 | 10 | 70 | 35 | 70 |
| 7 | 7 | 14 | 35 | 7 | 70 | 14 | 35 | 70 |
| 10 | 10 | 10 | 10 | 70 | 10 | 70 | 70 | 70 |
| 14 | 14 | 14 | 70 | 14 | 70 | 14 | 70 | 70 |
| 35 | 35 | 70 | 35 | 35 | 70 | 70 | 35 | 70 |
| 70 | 70 | 70 | 70 | 70 | 70 | 70 | 70 | 70 |

For Greatest Lower Bound (GLB) :

| | 1 | 2 | 5 | 7 | 10 | 14 | 35 | 70 |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 1 | 2 | 1 | 1 | 2 | 2 | 1 | 2 |
| 5 | 1 | 1 | 5 | 1 | 5 | 1 | 5 | 5 |
| 7 | 1 | 1 | 1 | 7 | 1 | 7 | 7 | 7 |
| 10 | 1 | 2 | 5 | 1 | 10 | 2 | 5 | 10 |
| 14 | 1 | 2 | 1 | 7 | 2 | 14 | 7 | 14 |
| 35 | 1 | 1 | 5 | 7 | 5 | 7 | 35 | 35 |
| 70 | 1 | 2 | 5 | 7 | 10 | 14 | 35 | 70 |

For every subset {a, b} consisting two elements, there exist a LUB and GLB. Hence it is a Lattice.

**Q.3(c) f : R → R is defined by $f(x) = x^3$** **[8]**
**g : R → R is defined by $g(x) = 4x^2 + 1$**
**h : R → R is defined by $h(x) = 7x - 2$**
**Find the rule defining**
**(i) fog (ii) gof (iii) (goh)of (iv) go(hof)**

**(A)** (i) $(fog)(x) = f\{g(x)\}$
$= f(4x^2 + 1)$
$= (4x^2 + 1)^3$

(ii) $(gof)(x) = g\{f(x)\} = g(x^3)$
$= 4(x^3)^2 + 1$

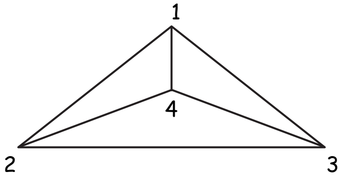
Note : fog $\neq$ gof

(iii) $\{(goh)of\}(x) = [(goh)\{f(x)\}]$
$= (goh)(x^3) = g\{h(x^3)\} = g(7x^3 - 2)$
$= 4(7x^3 - 2)^2 + 1$

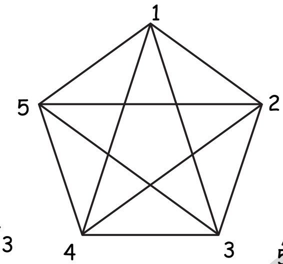
(iv) $\{go(hof)\}(x) = g\{(hof)(x)\}$
$= g[h\{f(x)\}] \quad = g[h(x^3)] \quad = g(7x^3 - 2)$
$= 4(7x^3 - 2)^2 + 1$

Note : $(goh)of = go(hof)$

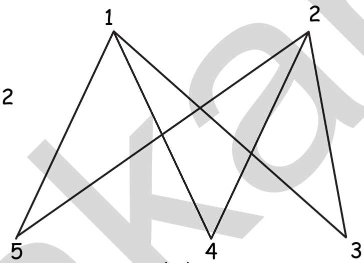
**Q.4(a) Decide which of the following graphs are Eulerian or Hamiltonian or both and write down as Eulerian circuit and Hamiltonian circuit wherever possible.** **[6]**



(i)



(ii)



(iii)

**(A)** (i) The degree of each vertex is odd.

$\therefore$ There is no Eulerian circuit and no Eulerian path.

No. of vertices = 4

Degree of each vertex = 3

$\because$ degree of each vertex $> \frac{1}{2} \times$ no. of vertices.

$\therefore$ There is a Hamiltonian circuit in a graph.

One Hamiltonian circuit is $1 \to 3 \to 2 \to 4 \to 1$

$\therefore$ Graph is Hamiltonian but not Eulerian.

(ii) The degree of each vertex is even.

$\therefore$ There is an Eulerian circuit in a graph.

One Eulerian circuit is $1 \to 2 \to 3 \to 4 \to 5 \to 3 \to 1 \to 4 \to 2 \to 5 \to 1$

$\therefore$ The graph is Eulerian.

Degree of each vertex = 4

Number of vertices = 5

$\because$ Degree of each vertex $> \frac{1}{2} \times$ no. of vertices.

$\therefore$ There is an Hamiltonian circuit in a graph.

One Hamiltonian circuit is $1 \to 2 \to 3 \to 4 \to 5 \to 1$

Hence the graph is Hamiltonian

Hence the graph is Hamiltonian & Eulerian.

**Note :** Every complete graph of odd number of vertices greater than or equal to 3 is both Eulerian as well as Hamiltonian.

(iii) There are two vertices of odd degree.

$\therefore$ There is an Eulerian path in the graph but no Eulerian circuit. Hence the graph is not Eulerian. An Eulerian path may begin with 1 and end on 2 and vice versa. One Eulerian path is $1\to5\to2\to4\to1\to3\to2$.

The graph has Hamiltonian path but no Hamiltonian circuit. One Hamiltonian path is $5\to1\to 4\to 2\to 3$

**Q.4(b) Let $G = \{0, 1, 2, 3, 4, 5\}$** **[6]**

**(i) Prepare composition table with respect to '$+_6$'**

**(ii) Prove that G is an abelian group with respect to '$+_6$'**

**(iii) Find the inverse of 2, 3 and 5.**

**(iv) Is it cyclic ?**

**(v) Find the order of 2, 3 and sub groups generated by these elements.**

**(A)** (i) Prepare composition table

| $+_6$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 3 | 4 | 5 |
| 1 | 1 | 2 | 3 | 4 | 5 | 0 |
| 2 | 2 | 3 | 4 | 5 | 0 | 1 |
| 3 | 3 | 4 | 5 | 0 | 1 | 2 |
| 4 | 4 | 5 | 0 | 1 | 2 | 3 |
| 5 | 5 | 0 | 1 | 2 | 3 | 4 |

(ii) (a) Closure axiom : $\because$ All the elements of C. T. $\in G$,

$\therefore$ it follows closure axiom.

(b) Associativity : $\forall$ a, b, c $\in G$

$a +_6 (b +_6 c) = (a +_6 b) +_6 c$ (from table)

(c) Identity : From table we find '0' is an identity element of G.

(d) Inverses : All the elements of G are invertible and their inverses are

$(0)^{-1} = 0$ $(3)^{-1} = 3$

$(1)^{-1} = 5$ $(4)^{-1} = 2$

$(2)^{-1} = 4$ $(5)^{-1} = 1$

(e) Commutativity : It follows commutative law

$\forall$ a, b, $\in G$, $a +_6 b = b +_6 a$

$\therefore$ G is an Abelian group with respect to '$+_6$'.

(iii) From table $2^{-1} = 4$, $3^{-1} = 3$, $5^{-1} = 1$

(iv) $1^1 = 1$

$1^2 = 1 +_6 1 = 2$

$1^3 = 1 +_6 1 +_6 1 = 3$

$1^4 = 1 +_6 1 +_6 1 +_6 1 = 4$

$1^5 = 1 +_6 1 +_6 1 +_6 1 +_6 1 = 5$

$1^6 = 1 +_6 1 +_6 1 +_6 1 +_6 1 +_6 1 = 0$

$\Rightarrow$ 1 is the generator of $G$

$\Rightarrow$ $G$ is a cyclic group

(v) $2^1 = 2$

$2^2 = 2 +_6 2 = 4$

$2^3 = 2 +_6 2 +_6 2 = 0$

$\therefore O(2) = 3$

$3^1 = 3$

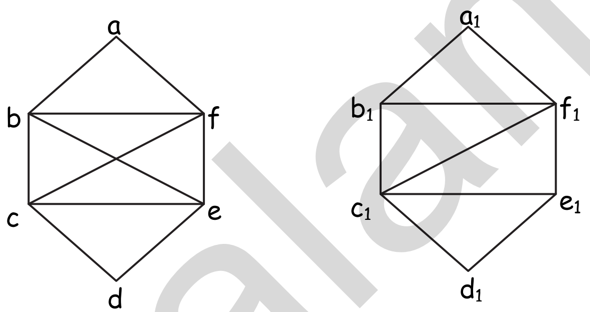
$3^2 = 3 +_6 3 = 0$

$\therefore O(3) = 2$

Subgroup generated by 2 is {0, 2, 4}

Subgroup generated by 3 is {0, 3}

**Q.4(c) Determine whether following graphs are isomorphic :** **[8]**




Graph $G_1$ Graph $G_2$

**(A)** (i) No. of vertices of $G_1$ = No. of vertices of $G_2$

$6 = 6$

(ii) No. of edges of $G_1$ = No. of edges of $G_2$

$8 \neq 9$

$\because$ No. of edges in $G_1$ and $G_2$ are not same.

$\therefore$ Graphs $G_1$ and $G_2$ are NOT isomorphic to each other.

**Q.5(a) Use the laws of logic to show that $[(p \to q) \wedge \sim q] \to \sim p$ is a tautology.** **[6]**

**(A)** $((p \to q) \wedge \sim q) \to \sim p$

$\equiv ((\sim p \vee q) \wedge \sim q) \to \sim p$

$\equiv ((\sim p \wedge \sim q) \vee (q \wedge \sim q)) \to \sim p$ … (Distributive law)

$\equiv ((\sim p \wedge \sim q) \vee F) \to \sim p$ … (Complement law)

$\equiv (\sim p \wedge \sim q) \to \sim p$ … (Identity law)

$\equiv \sim(\sim p \wedge \sim q) \vee \sim p$

$\equiv p \vee q \vee \sim p$

$\equiv (p \vee \sim p) \vee q$ … (Associative law)

$\equiv T \vee q$ ... (Complement law)

$\equiv T$ ... (Identity law)

Thus it is a tautology.

**Q.5(b) Let A = {1, 2, 3, 4} and R = {(1, 2), (2, 3), (3, 4), (2,1)}. Find the transitive closure using Warshalls Algorithm.** **[6]**

**(A)** Using Warshall's Algorithm :

$$w_0 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$C_1$ = 1's at position 2

$R_1$ = 1's at position 2

Put 1 at (2, 2)

$$w_1 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$C_2$ = 1's at position 1, 2

$R_2$ = 1's at position 1, 2, 3

Put 1 at (1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3)

$$w_2 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$C_3$ = 1's at position 1, 2

$R_3$ = 1's at position 4

Put 1 at (1, 4), (2, 4)

$$w_3 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$w_3 - w_4 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 1 & 1 & 1 \\ 2 & 1 & 1 & 1 & 1 \\ 3 & 0 & 0 & 0 & 1 \\ 4 & 0 & 0 & 0 & 0 \end{array}$$

. It is transitive closure of given relation.

**Q.5(c) Consider (2, 6) encoding function e : $B^2 \to B^6$ defined as** **[8]**

**e (00) = 000000     e (01) = 011110**

**e (10) = 101010     e (11) = 111000**

**(i) Find the minimum distance**

**(ii) How many error can 'e' detect ?**

**(A)** (2 < 6) and e : $B^2 \to B^6$ is an encoding function defined as

e (00) = 000000 = $x_0$

e (01) = 011110 = $x_1$

e (10) = 101010 = $x_2$

e (11) = 111000 = $x_3$

(i) $|x_0 \oplus x_1| = |011110| = 4$ , $|x_0 \oplus x_2| = |101010| = 3$

$|x_0 \oplus x_3| = |111000| = 3$, $|x_1 \oplus x_2| = |110100| = 3$

$|x_1 \oplus x_3| = |100110| = 3$, $|x_2 \oplus x_3| = |010010| = 2$

$\Rightarrow$ Minimum distance = 2

(ii) $\because$ An (m, n) encoding function can detect k or fewer error if the minimum distance is (k + 1)

k + 1 = 2

k = 1

$\Rightarrow$ e can detect 1 or fewer error.

**Q.6(a) Find formula for sequences with following first five terms.** **[6]**

**(a) 1, 1/2, 1/4, 1/8, 1/16**

**(b) 1, 3, 5, 7, 9**

**(c) 1, −1, 1, −1, 1**

**(A)** To find formula,

(a) $\frac{1}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16} = \frac{1}{2^0}, \frac{1}{2^1}, \frac{1}{2^2}, \frac{1}{2^3}, \frac{1}{2^4}$

$= \sum_{n=0}^{4} \frac{1}{2^n}$

(b) 1, 3, 5, 7, 9

This is arithmetic progression,

Here a = 1, d = 2

Now, $t_n = a + (n - 1)d$

$= 1 + (n - 1) 2 = 2n - 1$

$\therefore \quad t_n = \sum_{n=1}^{5}(2n-1)$

(c) 1, –1, 1, –1, 1

alternate positive, negative term exist,

$\therefore \quad t \quad = \sum_{r=0}^{4}(-1)^r$

**Q.6(b) A connected planar graph has 9 vertices having degrees 2, 2, 2, 3, 3, 3, 4, 4 and 5. How many edges are there? [6]**

**(A)** For any graph G, the sum of degrees of vertices of G is twice of the no. of edges.

$\therefore$ 2 + 2 + 2 + 3 + 3 + 3 + 4 + 4 + 5 = 2E

$\therefore$ 2E = 28

$\therefore$ E = 14

$\Rightarrow$ No. of edges are 14.

**Q.6(c) Determine the sequence whose recurrence relation is given by $C_n = 3C_{n-1} - 2C_{n-2}$ with initial conditions $C_1 = 5$, $C_2 = 3$. [8]**

**(A)** The quadratic equation associated with this recurrence relation is $x^2 = 3x - 2$ with every linear homogenous recurrence relation there is associated a quadratic equation.

$\therefore \quad C_n = 3C_{n-1} - 2C_{n-2}$

$\therefore \quad x^2 = 3x - 2$

$\therefore \quad x^2 - 3x + 2 = 0$

$\therefore \quad (x - 1)(x - 2) = 0$

$\Rightarrow \quad x = 1, 2$

$\therefore \quad C_n = u(1)^n + v(2)^n$

$\therefore \quad C_n = u + v(2)^n$

Put n = 1

$\therefore \quad C_1 = u + 2v$

$\therefore \quad u + 2v = 5$ … (1)

Put n = 2

$\therefore \quad C_2 = u + 4v$

$\therefore \quad u + 4v = 3$ … (2)

$\therefore \quad v = -1$ & $u = 7$

$\therefore \quad C_n = 7 - (2)^n$

$\therefore$ Sequence is 5, 3, –1, –9, …

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