S.E. Sem. III [EXTC]

Applied Mathematics - III

Time : 3 Hrs.] Prelim Question Paper **[Marks : 80**

N.B.: (1) Question No. **1** is **compulsory**.
(2) Attempt any **THREE** of the remaining.
(3) Figures to the right indicate full marks.

1. (a) Find Laplace Transformation of $\frac{\sin(3t)}{t}$ **[5]**

(b) Prove that f(z) = cosh z is analytic and find its derivate. **[5]**

(c) Find Fourier series for $f(x) = 16 - x^2$ over (−4, +4) **[5]**

(d) Prove that $J_{\frac{3}{2}}(x) = \sqrt{\frac{2}{\pi x}}\left(\frac{\sin(x)}{x} - \cos(x)\right)$ **[5]**

2. (a) Show that the vector field $\bar{F} = (3x^2 y)i + (x^3 - 2yz^2)j + (3z^2 - 2y^2z)k$ is conservative and find $\phi(x, y, z)$ such that $\bar{F} = \nabla\phi$. Also evaluate the line integral $\int \bar{F}\cdot d\bar{r}$ from (2, 1, 1) to (2, 0, 1) **[6]**

(b) Find the Fourier series for $f(x) = \frac{x-\pi}{4}$; $0 \le x \le 2\pi$. Hence prove that **[6]**

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots = \frac{\pi}{4}$$

(c) Find Inverse Laplace transform of **[8]**

i) $\frac{s+19}{(s+9)(s^2+4)}$, ii) $\frac{e^{-3s}}{(s^2+10s+29)}$.

3. (a) Find the analytic function f(z) = u + iv if $u + v = \frac{2\sin(2x)}{e^{2y} + e^{-2y} - 2\cos(2x)}$ **[6]**

(b) Show that Bessel's Fourier series in $J_2(\lambda_n x)$ for $f(x) = x^2$, $(0 < x < a)$ Where λ_n positive root of $J_2(ax) = 0$ is $x^2 = 2a\sum_{n=1}^{\infty}\frac{J_2(\lambda_n x)}{\lambda_n J_3(\lambda_n a)}$. **[6]**

(c) Solve the differential equation $\frac{d^2y}{dt^2} + 4y = f(t)$, with y(0) = 0 & $y'(0) = 1$ and $f(t) = \begin{matrix} 1 & 0<t<1 \\ 0 & 1<t \end{matrix}$. **[8]**

4. (a) Find the orthogonal Trajectory of $3x^2 - 2x^2y + y^2 = \text{cont.}$ **[6]**

(b) Using Greens theorem **[6]**

Evaluate $\oint_C \left((2xy)dx - (y^2)dy\right)$ where C is boundary of the region bounded by.

(c) Find the Fourier integral representation of $f(x) = \begin{matrix} 1-x^2 & \text{for} |x| \le 1 \\ 0 & \text{for} |x| > 1 \end{matrix}$. **[8]**

Hence prove that $\int_{\lambda=0}^{\infty} \frac{\cos\left(\frac{\lambda}{2}\right)(\sin\lambda - \lambda\cos\lambda)}{\lambda^3} d\lambda = \frac{3\pi}{16}$.

5. (a) Find inverse Laplace Transform of $\frac{s}{s^4 + 8s^2 + 16}$ using Convolution theorem. **[6]**

(b) Find the Bilinear Transform which transform the points z = 2, i, –2 of z-plane into the points w = 1, i, –1 of the w-plane respectively. Also find fixed points of this transformation. **[6]**

(c) Find $\iint_s \left[(\nabla \times \bar{F}) \cdot \hat{n}\right] ds$ **[8]**

$\bar{F} = (2x - y + z)i + (x + y - z^2)j + (3x - 2y + 4z)k$ over the surface of the cylinder $x^2 + y^2 = 4$ bounded by z = 9 and open at the end z = 0.

6. (a) Find the directional derivative of $\phi = xy^2 + yz^2$ at (2, –1, 1) along the line $2(x - 2) = y + 1 = z - 1$. **[6]**

(b) Obtain the complex form of Fourier series for the function $f(x) = e^{4x}$ in $0 < x < 4$. **[6]**

(c) Find half range sine series of the function $f(x) = x(3 - x)$ in $0 \le x \le 3$ **[8]**

Hence prove that :

(i) $\frac{1}{1^6} + \frac{1}{3^6} + \frac{1}{5^6} + \frac{1}{7^6} + \ldots = \frac{\pi^6}{960}$ (ii) $\sum_{n=1}^{\infty} \frac{1}{(n)^6} = \frac{\pi^6}{945}$

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