Circuits Theory

Time : 3 Hrs. Prelim Question Paper Solution **[Marks : 80**

Q.1(a) Test whether $P(s) = s^5 + 12s^4 + 45s^3 + 60s^2 + 44s + 48$ is Hurwitz polynomial. **[5]**

(A) $P(s) = s^5 + 12s^4 + 45s^3 + 60s^2 + 44s + 48$

s^5	1	45	44
s^4	12	60	48
s^3	40	40	0
s^2	48	48	
s^1	0	0	

As complete row becomes zero we will form an auxiliary equation.

$A(s) = 48s^2 + 48$

$A'(s) = 96s + 0$

Replacing zeroth row with $A'(s)$

s^5	1	45	44
s^4	12	60	48
s^3	40	40	0
s^2	48	48	
s^1	96	0	
s^0	48		

As all the elements in the 1st columns are positive given f^n is a Hurwitz polynomial.

Q.1(b) The constants of a transmission line are R = 6 Ω/km, L = 2.2 mH/km, G = 0.25 × 10^{-6} ℧/km, C = 0.005 × 10^{-5} F/km. Determine the characteristic impedance, propagation constant and attenuation constant at 1 KHz. **[5]**

(A) Given : $R = 6\Omega$ / km, L = 2.2 mH/km, $G = 0.25 \times 10^{-6}$ ℧ / km

$C = 0.005 \times 10^{-5}$ F/km, F = 1KHz

To find : Zo, γ, α

$z = R + j\omega L$

$= 6 + j2\pi \times 10^3 \times 2.2 \times 10^{-3}$

$= 6 + j13.82$

$z = 15.06 \angle 66.53^\circ$

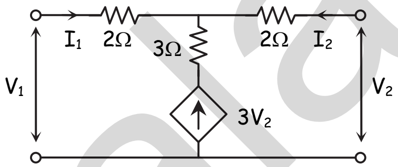
$y = G + j\omega C$
$= 0.25 \times 10^{-6} + j2\pi\, 10^{3} \times 0.005 \times 10^{-5}$
$= 0.25 \times 10^{-6} + j\, 3.14 \times 10^{-4}$
$= 3.14 \times 10^{-4} \angle 89.95^\circ$

Characteristic impedance $= z_o = \sqrt{\dfrac{z}{y}} = \sqrt{\dfrac{15.06\angle 66.53^\circ}{3.14\times 10^{-4}\angle 89.95^\circ}}$
$= 219.00 \angle 11.71^\circ\Omega$

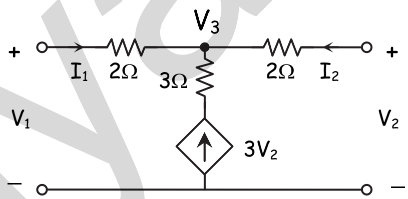
Propagation constant

$\gamma = \sqrt{z.y} = \sqrt{(15.06\angle 66.53)(3.14\times 10^{-4}\angle 89.95)}$
$\gamma = 0.014 + j\, 0.067$ / km
$\gamma = \alpha + j\beta$
$\alpha = 0.014$ Np/km,
$\beta = 0.067$ rad/km

Q.1(c) Determine the short circuit admittance parameters of the network shown : **[5]**



(A)



Apply RCL at V_3

$$\frac{V_3 - V_1}{2} + \frac{V_3 - V_2}{2} = 3V_2$$

$$V_3\left(\frac{1}{2} + \frac{1}{2}\right) = 3V_2 + \frac{V_2}{2} + \frac{V_1}{2}$$

$$V_3 = 0.5\, V_1 + 3.5\, V_2 \qquad \text{...(1)}$$

Apply KCL at V_1

$$I_1 = \frac{V_1 - V_3}{2}$$

$$I_1 = \frac{V_1}{2} = \frac{1}{2}\left(0.5V_1 + 3.5\,V_2\right)$$

$I_1 = V_1(0.25) - V_2(1.75)$

Comparing with $I_1 = y_{11}V_1 + y_{12}V_2$

$y_{11} = 0.25\,℧$ $\quad$ $y_{12} = -1.75\,℧$

Apply KCL at V_2

$$I_2 = \frac{V_2 - V_3}{2}$$

$$I_2 = \frac{V_2}{2} - \frac{1}{2}(0.5V_1 + 3.5V_2)$$

$$I_2 = -0.25\,V_1 - 1.25\,V_2$$

$y_{21} = -0.25\,℧$, $\quad$ $y_{22} = -1.25\,℧$

Q.1(d) State and prove final value theorem of Laplace transform. **[5]**

(A) If L {f(t)} = F(s) then $\lim_{t\to 0} f(t) = \lim_{s\to 0} sF(s)$

Proof : We know that

$$L\{f'(t)\} = s\,F(s) - f(0)$$

$$s\,F(s) = L\{f'(t)\} + f(0)$$

$$s\,F(s) = \lim_{s\to 0}\int_0^\infty f'(t)e^{-st}\,dt + f(0)$$

$$= \int_0^\infty \lim_{s\to 0}[f'(t)e^{-st}]dt + f(0)$$

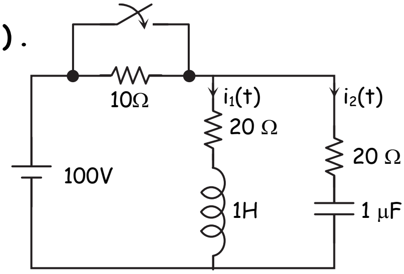
$$= \int_0^\infty f'(t)\,dt + f(0)$$

$$= |f(t)|_0^\infty + f(0)$$

$$= \lim_{t\to\infty} f(t) - f(0) + f(0)$$

$$= \lim_{t\to\infty} f(t)$$

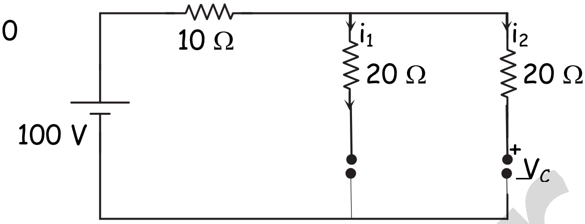
Q.2(a) The network shown in figure, a steady state is reached with the switch open. At t = 0, the switch is closed. Determine $V_c(0^-)$, $i_1(0^+)$, $i_2(0^+)$, $\frac{di_1}{dt}(0^+)$ and $\frac{di_2}{dt}(0^+)$. **[10]**



(A) **Step 1:** At $t = 0^-$, switch is open

$i_2(0^-) = 0$

finding $i_1(0^-)$



$$100 - 10\,i_1 - 20\,i_1 = 0$$

$$100 = 30\,i_1$$

$$\therefore \quad i_1 = \frac{100}{30} = \frac{10}{3}A$$

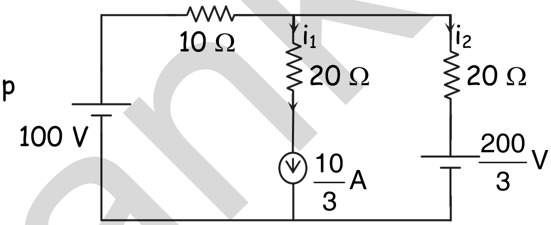
$$\therefore \quad i_1(0^-) = \frac{10}{3}A$$

$$V_C(0^-) = 20 \times i_1(0^-) = 20 \times \frac{10}{3}$$

$$V_C(0^-) = \frac{200}{3}V$$

Step 2: At $t = 0^+$

$$i_1(0^+) = \frac{10}{3}A$$



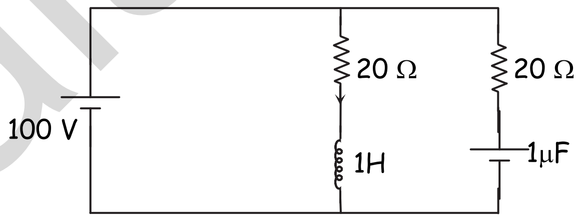
Apply KVL to the outer loop

$$100 - 20i_2 - \frac{200}{3} = 0$$

$$20\,i_2 = 100 - \frac{200}{3}$$

$$i_2(0^+) = 1.67\text{ A}$$

Step 3: At $t > 0$



Apply KVL to loop (1)

$$100 - 20\,i_1 - \frac{di_1}{dt} = 0$$

$$\frac{di_1}{dt} = 100 - 20\,i(0^+)$$

$$= 100 - \frac{20 \times 10}{3}$$

$$\frac{di_1}{dt} = 33.33\text{ A/s}$$

Apply KVL to outer loop

$$100 - 20\,i_2 - \frac{1}{10^{-6}}\int i_2 dt = 0$$

Differentiate w.r.t. t

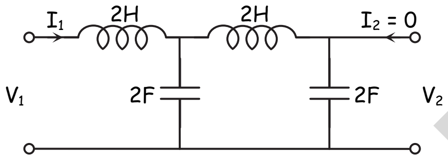
$$0 - 20\frac{di_2}{dt} - 10^6\,i_2 = 0$$

$$-20\frac{di_2(0^+)}{dt} = 10^6\,i_2(0^+)$$

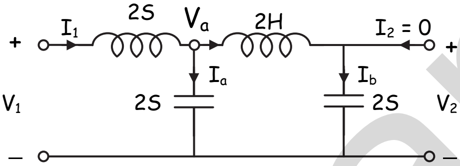
$$\frac{di_2}{dt} = \frac{-10^6(1.67)}{20}$$

$$\frac{di_2}{dt} = -83500\ A/s$$

Q.2(b) Find the network functions $\frac{V_1}{I_1}, \frac{V_2}{I_1}, \frac{V_2}{V_1}$ for the network shown : **[5]**



(A)



Let $V_2 = 1$

$I_b = V_2 \times 2S$

$V_a = V_2 + I_b \cdot 2S$

$= 1 + 2S \cdot 2S$

$V_a = 1 + 4S^2$

$I_a = V_a \cdot 2S$

$= (1 + 4S^2)2S$

$I_a = 2S + 8S^3$

$I_1 = I_a + I_b$

$= 2S + 8S^3 + 2S$

$I_1 = 8S^3 + 4S$

$V_1 = V_a + I_i 2S$

$= 1 + 4S^2 + (8S^3 + 4S)2S$

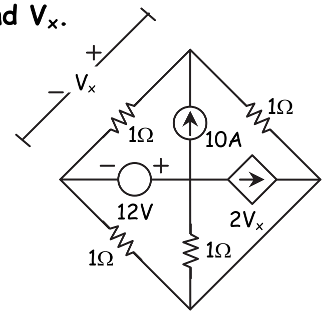
$= 1 + 4S^2 + 16S^4 + 8S^2$

$V_1 = 16S^4 + 12S^2 + 1$

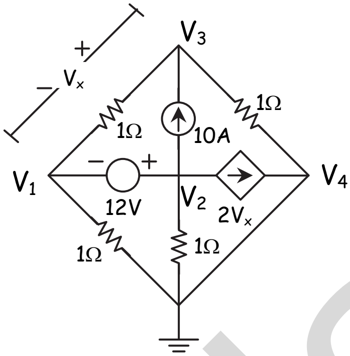
$$\frac{V_1}{I_1} = \frac{16S^4 + 12S^2 + 1}{8S^3 + 4S}$$

$$\frac{V_2}{I_1} = \frac{1}{8S^3 + 4S} \quad \& \quad \frac{V_2}{V_1} = \frac{1}{16S^4 + 12S^2 + 1}$$

Q.2(c) In the circuit shown in figure, find V_x. **[5]**



(A)



$V_4 = 0$ V

$V_x = V_1 - V_3$ ...(1)

Relation between V_1 & V_2

$V_2 - V_1 = 12$

$-V_1 + V_2 + 0V_3 = 12$ ...(2)

Supernode equation

$$\frac{V_1 - V_3}{1} + \frac{V_1}{1} + \frac{V_2}{1} + 10 + 2V_x = 0$$

$V_1 - V_3 + V_1 + V_2 + 10 + 2(V_1 - V_3) = 0$ ...(from 1)

$4V_1 + V_2 - 3V_3 = -10$ ...(3)

KCL to V_3

$$\frac{V_3 - V_1}{1} + \frac{V_3}{1} = 10$$

$-V_1 + 0V_2 + 2V_3 = 10$ ...(4)

Solving (2), (3) & (4)

$V_1 = -2$ V

$V_2 = 10$ V

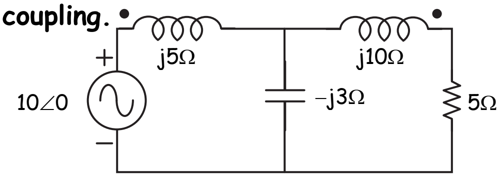
$V_3 = 4$ V

$V_x = V_1 - V_3$

$V_x = -2 - 4$

$V_x = -6$ V

Q.3(a) Find the voltage across 5 Ω resistor in the network shown below. If K = 0.8 is coefficient of coupling. **[8]**

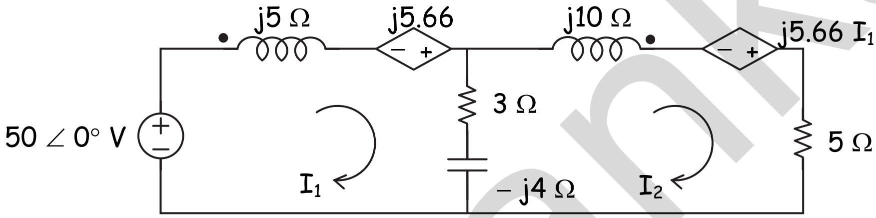


(A) For a magnetically coupled circuit,

$M = K\sqrt{5(10)}$

$X_m = K\sqrt{X_{L_1} X_{L_2}} = 0.8\sqrt{5(10)} = 5.66\ \Omega$

The equivalent circuit in terms of dependent sources can be drawn as



Applying KVL to Mesh 1,

$50\angle 0° - j5I_1 - 3(I_1 - I_2) + j4(I_1 - I_2) + j5.66I_2 = 0$

$50\angle 0° = (3 + j1)I_1 - (3 + j1.66)I_2$ ... (1)

$(3 + j1)I_2 + (-3 - j1.66)I_2 = 50\angle 0°$

Applying KVL to Mesh 2,

$j4(I_2 - I_1) - 3(I_2 - I_1) - j10I_2 + j5.66I_1 - 5I_2 = 0$

$j4I_2 - j4I_1 - 3I_2 + 3I_1 - j10I_2 + j5.66I_1 - 5I_2 = 0$

$-j4I_2 + j4I_1 + 3I_2 - 3I_1 + j10I_2 - j5.66I_1 + 5I_2 = 0$

$(-3 - j1.66)I_1 + (8 + j6)I_2 = 0$ ... (2)

By Cramer's rule,

$$I_2 = \frac{\begin{vmatrix} 3+j1 & 50\angle 0° \\ -3-j1.66 & 0 \end{vmatrix}}{\begin{vmatrix} 3+j1 & -3-j1.66 \\ -3-j1.66 & 8+j6 \end{vmatrix}} = 8.62\angle -24.79°\ A$$

$V = 5I_2 = 5(8.62\angle -24.79°) = 43.1\angle -24.79°\ V$

Q.3(b) Check the positive real function : **[8]**

(i) $F(s) = \dfrac{s^2 + 6s + 5}{s^2 + 9s + 14}$ **(ii)** $F(s) = \dfrac{s^3 + 6s^2 + 7s + 3}{s^2 + 2s + 1}$

(A) (i) $F(s) = \dfrac{s^2 + 6s + 5}{s^2 + 9s + 14}$

Step 1 : Let $F(s) = \dfrac{N(s)}{D(s)} = \dfrac{s^2 + 6s + 5}{s^2 + 9s + 14}$

$$F(s) = \frac{(s+5)(s+1)}{(s+7)(s+2)}$$

The function f(s) has poles at s = –7 & s = –2 & zeros at s = –5 and s = –1

$\therefore$ All the poles & zeros are in the left half of s–plane.

Step 2 : As there is no pole on jw axis, the residue test is not carried out.

Step 3 : Even part of N(s) = $m_1 = s^2 + 5$

Odd part of N(s) = $n_1 = 6s$

Even part of D(s) = $m_2 = s^2 + 14$

Odd part of D(s) = $n_2 = 9s$

$A(w^2) = m_1m_2 - n_1n_2 \quad = (s^2 + 5)(s^2 + 14) - (6s)(9s)\,|_{s = jw}$

$= s^4 - 35s^2 + 70\,|_{s = jw}$

$\therefore$ $A(w^2) = w^4 + 35w^2 + 70$

$A(w^2)$ is positive for all $w \geq 0$

As all the three conditions are satisfied, the function is a positive real function.

(ii) $F(s) = \dfrac{s^3 + 6s^2 + 7s + 3}{s^2 + 2s + 1}$

Step 1 : Let $F(s) = \dfrac{N(s)}{D(s)} = \dfrac{s^3 + 6s^2 + 7s + 3}{s^2 + 2s + 1}$

$$F(s) = \frac{s^3 + 6s^2 + 7s + 3}{(s+1)^2}$$

From above both the poles are lying at s = –1

Now we will check for N(s)

$N(s) = s^3 + 6s^2 + 7s + 3$

The Routh array of N(s) is given by

s^3	1	7
s^2	6	3
s^1	$\dfrac{39}{6}$	0
s^0	3	

As all elements in 1^{st} column are +ve, N(s) is Hurwitz polynomial.

$\therefore$ 1^{st} condition is satisfied.

Step 2 : As there is no pole on jw axis, residue test need not to be carried out.

Step 3 : Even part of N(s) = $m_1 = 6s^2 + 3$
Odd part of N(s) = $n_1 = s^3 + 7s$
Even part of D(s) = $m_2 = s^2 + 1$
Odd part of D(s) = $n_2 = 2s$

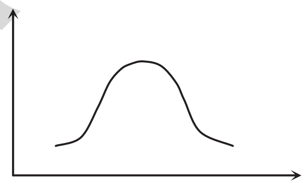
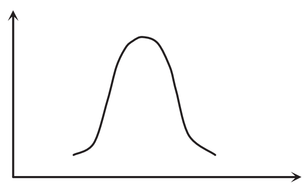
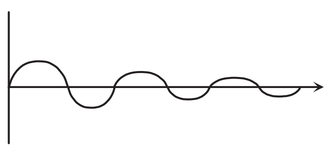
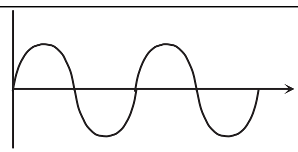
$A(w^2) = m_1m_2 - n_1n_2$
$= (6s^2 + 3)(s^2 + 1) - (s^3 + 7s)(2s) \,|_{s=jw}$
$= 4s^4 - 5s^2 + 3 \,|_{s = jw}$
$A(w^2) = 4w^4 + 5w^2 + 3$

$\therefore$ We can say that $A(w^2)$ is positive for all $w \geq 0$

As all three conditions are satisfied, function f(s) is a positive real function.

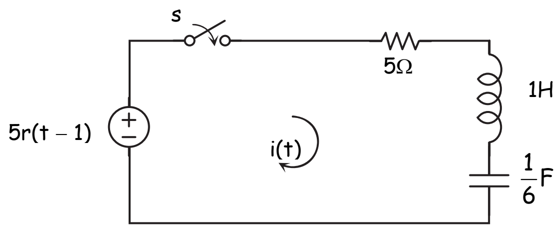
Q.3(c) List the types of damping in series R-L-C circuit and mention the condition for each damping. **[4]**

(A)

	Nature of Roots	System	Response
1.	Negative Real unequal	Over damped $k_1e^{p_1t} + k_2e^{p_2t}$	
2.	Negative Real equal	Critically damped $k_1e^{pt} + k_2te^{pt}$	
3.	Complex conjugate	Underdamped $e^{\sigma t}\left[k_1 \cos\omega t + k_2 \sin\omega t\right]$	
4.	Conjugate imaginary	Oscillatory $k_1 \cos\omega t + k_2 \sin\omega t$	

where $\alpha = \dfrac{R}{2L}$ and $\omega_0 = \dfrac{1}{\sqrt{LC}}$

Q.4(a) For the network shown, determine the current i(t) when the switch is closed at t = 0 with zero initial conditions. **[8]**

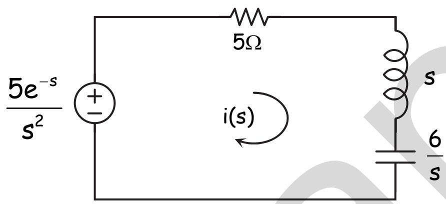


(A) Step 1 At $t = 0^-$, switch is open

$i(0^-) = 0$ A

$V_c(0^-) = 0V$

Step 2 At t > 0, switch is closed.



Applying KVL

$$5\frac{e^{-s}}{s^2} - 5\,i(s) - s\,i(s) - \frac{6}{s}i(s) = 0$$

$$\frac{5e^{-s}}{s^2} = i(s)\left(5 + s + \frac{6}{s}\right)$$

$$\frac{5e^{-s}}{s^2} = \frac{5s + s^2 + 6}{s}\ i(s)$$

$$(s^2 + 5s + 6)\ i(s) = \frac{5e^{-s}}{s}$$

$$i(s) = \frac{5e^{-s}}{s(s^2 + 5s + 6)}$$

$$i(s) = \frac{5e^{-s}}{s(s+3)(s+2)}$$

Let $i_1(s) = \frac{1}{s(s+3)(s+2)} = \frac{A}{s} + \frac{B}{s+3} + \frac{C}{s+2}$

$1 = A(s + 3)(s + 2) + B\,s(s + 2) + C\,s(s + 3)$

$A = \frac{1}{6}$, $B = \frac{1}{3}$, $C = -\frac{1}{2}$

$i(C) = 5e^{-s} \times i_1(s)$

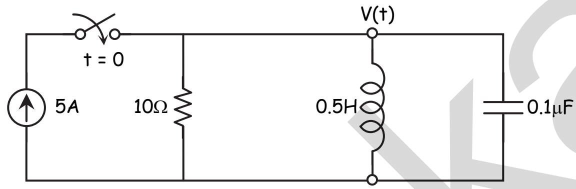
$$= 5e^{-s}\left[\frac{1}{6s} + \frac{1}{3(s+3)} - \frac{1}{2(s+2)}\right]$$

$$i(s) = \frac{5e^{-s}}{6s} + \frac{5\,e^{-s}}{3(s+3)} - \frac{5e^{-s}}{2(s+2)}$$

Taking inverse laplace transform

$$i(t) = \frac{5}{6}u(t-1) + \frac{5}{3}e^{-3(t-1)}\,u(t-1) - \frac{5}{2}e^{-2(t-1)}\,u(t-1)$$

Q.4(b) In the given network switch is closed at t = 0. Solve for V, $\frac{dV}{dt}$, $\frac{d^2V}{dt^2}$ at t = 0⁺. **[8]**

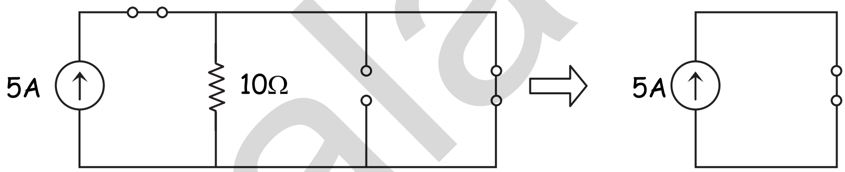


(A) Step 1: At $t = 0^-$, switch is open.

$i_L(0^-) = 0$ A

$V_c(0^-) = 0V$ & $V(0^-) = 0V$

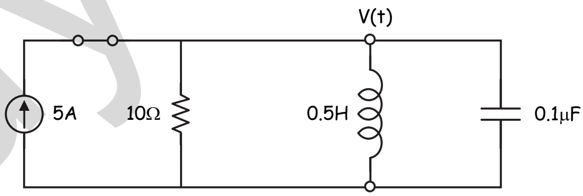
Step 2: At $t = 0^+$, switch is closed.



$i_L(0^+) = 0A$

$V_1(0^+) = 0V$, $V(0^+) = 0V$

Step 3: At t > 0



Applying KCL to V

$$\frac{V}{10} + \frac{1}{0.5}\int V\,dt + 0.1\times10^{-6}\frac{dV}{dt} = 5 \qquad \text{...(1)}$$

At $t = 0^+$

$$\frac{V(0^+)}{10} + 0 + 0.1\times10^{-6}\frac{dV}{dt}(0^+) = 5$$

$$0.1\times10^{-6}\frac{dV}{dt}(0^+) = 5 - 0$$

$$\frac{dV}{dt}(0^+) = \frac{5}{0.1\times10^{-6}}$$

$$\frac{dV}{dt}(0^+) = 50 \times 10^{+6} \text{ v/s}$$

Diff. equation (1) w.r.t. t

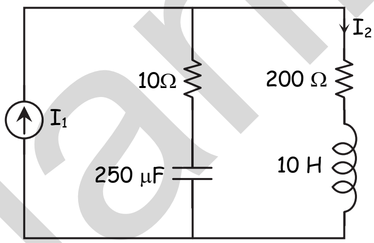
$$\frac{dV}{10\,dt} + \frac{1}{0.5}\ V(0^+) + 0.1 \times 10^{-6}\ \frac{d^2V}{dt^2} = 0$$

$$0.1\times10^{-6}\frac{d^2V}{dt^2} = \frac{-dV}{10dt}$$

$$\frac{d^2V}{dt^2} = -\frac{50\times10^6}{10\times0.1\times10^{-6}}$$

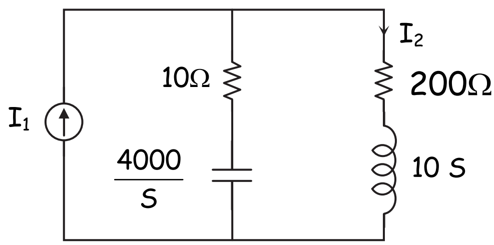
$$\frac{d^2V}{dt^2} = -50 \times 10^{12}\ V/s^2$$

Q.4(c) Obtain pole-zero plot for $\frac{I_2}{I_1}$. **[4]**



(A) By C.D.R.

$$I_2 = \frac{\frac{4000}{S}+10}{\frac{4000}{S}+10+200+10S}\times I_1$$



$$\frac{I_2}{I_1} = \frac{\frac{4000+10S}{S}}{\frac{4000+10S+200S+10S^2}{S}}$$

$$\frac{I_2}{I_1} = \frac{4000+10S}{10S^2+210S+4000}$$

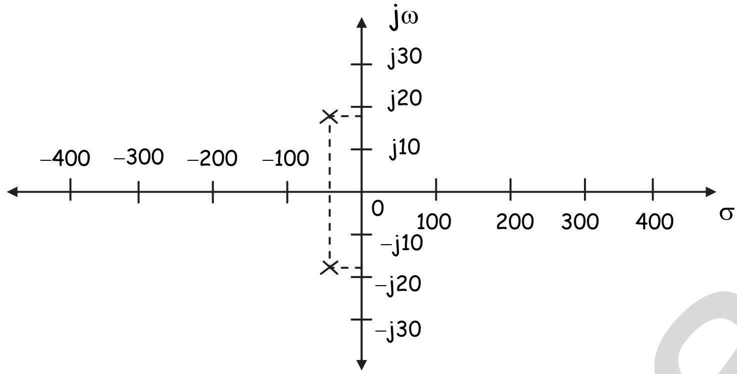
$$\frac{I_2}{I_1} = \frac{10(S+400)}{10\,(S^2+21S+400)}$$

$$\frac{I_2}{I_1} = \frac{S+400}{S^2+21S+400}$$

$Z_1 = -400$

$P_1 = -10.5 + 17.02j$

$P_2 = -10.5 - 17.02j$



Q.5(a) Synthesize the driving point function using Foster-I and Foster-II form : **[10]**

$$Z(s) = \frac{2(s^2+1)(s^2+9)}{s(s^2+4)}$$

(A) Foster – I form :

A degree of numerator is greater than degree of denominator, division is first carried out.

$$Z(s) = \frac{4(s^2++1)(s^2+9)}{s(s^2+4)} = \frac{4s^4+40s^2+36}{s^3+4s}$$

$$\begin{array}{r|l} & 4s \\ s^3+4s & 4s^4+40s^2+36 \\ & \underline{4s^4+16s^2} \\ & 24s^2+36 \end{array}$$

$$\therefore \quad Z(s) = \text{Quotient} + \frac{\text{Remainder}}{\text{Diviser}} = 4s+\frac{24s^2+36}{s^3+4s} = 4s+\frac{24s^2+36}{s(s^2+4)}$$

By partial fraction expansion

$$Z(s) = 4s+\frac{k_0}{s}+\frac{2k_1 s}{s^2+4}$$

$$k_0 = sz(s)\big|_{s=0} = \frac{4(1)(9)}{4} = 9$$

$$k_1 = \left.\frac{(s^2+4)z(s)}{2s}\right|_{s^2=-4} = \frac{4(-4+1)(-4+9)}{2(-4)} = \frac{15}{2}$$

$$Z(s) = 4s+\frac{9}{s}+\frac{15s}{s^2+4}$$

1st term $\Rightarrow$ $4s$ $\Rightarrow$ inductor of 4H

2nd term $\Rightarrow$ $\frac{9}{s}$ $\Rightarrow$ capacitor of $\frac{1}{9}$F

3rd term $\Rightarrow$ parallel LC network

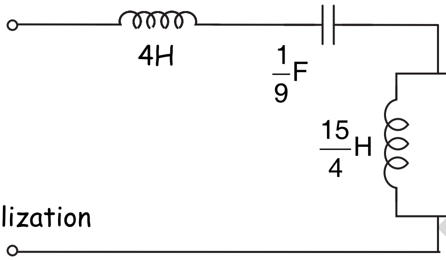
For parallel LC network

$$Z_{LC}(s) = \frac{\left(\frac{1}{c}\right)_s}{s^2 + \frac{1}{LC}}$$

Comparing the terms

$\therefore \quad C = \frac{1}{15}F$ and $L = \frac{15}{4}H$

$\therefore$ Corresponding network is as follows.



F–I
Realization

Foster II form :

It is obtain by partial fraction expansion of admittance function

$$Y(s) = \frac{s(s^2+4)}{4(s^2+1)(s^2+9)}$$

By partial fraction expansion

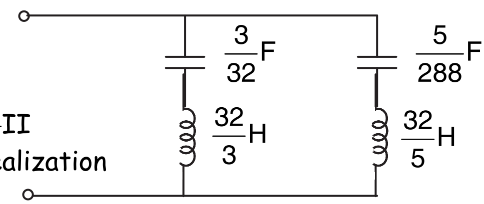
$$Y(s) = \frac{2k_1 s}{s^2+1} + \frac{2k_2 s}{s^2+9}$$

$$k_1 = \frac{(s^2+1)}{2s} y(s)\Big|_{s^2=-1} = \frac{(-1+4)}{8(-1+9)} = \frac{3}{64}$$

$$k_2 = \frac{s^2+9}{2s} y(s)\Big|_{s^2=-9} = \frac{(-9+4)}{8(-9+1)} = \frac{5}{64}$$

$$y(s) = \frac{\left(\frac{3}{32}\right)s}{s^2+1} + \frac{\left(\frac{5}{32}\right)s}{s^2+9}$$

Above two terms presents admittance of series LC network for series LC network $\Rightarrow Y_{LC}(s) = \frac{\left(\frac{1}{L}\right)s}{s^2 + \frac{1}{LC}}$



F–II
Realization

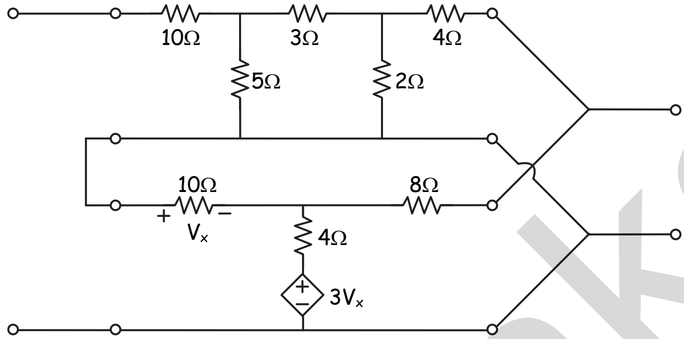
$\therefore$ By comparison

$L_1 = \frac{3L}{3}H \qquad C_1 = \frac{3}{32}F$

$L_2 = \frac{32}{5} H \qquad C_2 = \frac{5}{288} F$

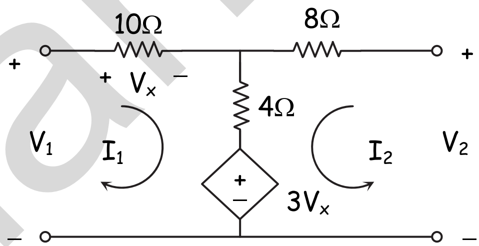
Corresponding network is as shown.

Q.5(b) Obtain hybrid parameter of the inter-connected network. [10]



(A) Separate the two networks

1^{st} network



$V_x = 10\, I_1$

Apply KVL to loop of I_1

$V_1 - 10I_1 - 4(I_1 + I_2) - 3V_x = 0$

$V_1 = 10\, I_1 + 4(I_1 + I_2) + 3(10I_1)$

$V_1 = 10I_1 + 4I_1 + 4I_2 + 30I_1$

$V_1 = 44I_1 + 4I_2$

Comparing with $V_1 = z_{11}\, I_1 + z_{12} I_2$

$\therefore \quad z_{11} = 44\Omega \qquad z_{12} = 4\Omega$

Apply KVL to loop of I_2

$V_2 - 8I_2 - 4(I_2 + I_1) - 3V_x = 0$

$V_2 = 8I_2 + 4(I_2 + I_1) + 3\,(10I_1)$

$V_2 = 34I_1 + 12I_2$

Comparing with

$V_2 = z_{21} I_1 + z_{22}\, I_2$

$\therefore \quad z_{21} = 34\ \Omega \qquad z_{22} = 12\Omega$

$\Delta z = z_{11}\, z_{22} - z_{21}\, z_{12} = 44 \times 12 - 34 \times 4$

$\Delta z = 392$

Now finding h-parameters from Z-parameter

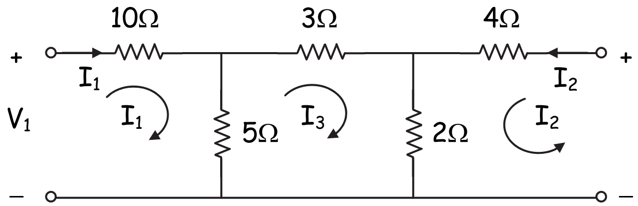
$h'_{11} = \frac{\Delta z}{z_{22}} = \frac{392}{12} = 32.66$

$h''_{12} = \frac{z_{12}}{z_{22}} = \frac{4}{12} = 0.333$

$h'_{21} = \frac{-z_{21}}{z_{22}} = \frac{-34}{12} = -2.833$

$$h'_{22} = \frac{1}{z_{22}} = \frac{1}{12} = 0.0833$$

$$\therefore \begin{bmatrix} h'_{11} & h'_{12} \\ h'_{21} & h'_{22} \end{bmatrix} = \begin{bmatrix} 32.66 & 0.333 \\ -2.833 & 0.0833 \end{bmatrix}$$



Apply KVL to I_3

$$-5(I_3 - I_1) - 3I_3 - 2(I_3 + I_2) = 0$$

$$-10I_3 + 5I_1 - 2I_2 = 0$$

$$I_3 = \frac{-5I_1 + 2I_2}{-10}$$

$$I_3 = 0.5\, I_1 - 0.2\, I_2 \qquad \text{...(1)}$$

Apply KVL to I_1

$$V_1 - 10I_1 - 5(I_1 - I_3) = 0$$

$$V_1 = 10I_1 + 5I_1 - 5(0.5I_1 - 0.2I_2)$$

$$V_1 = 12.5I_1 + I_2$$

$\therefore$ $Z_{11} = 12.5\Omega$ & $Z_{12} = 1\Omega$

Apply KVL to I_2

$$V_2 - 4I_2 - 2(I_2 + I_3) = 0$$

$$V_2 = 4I_2 + 2I_2 + 2(0.5I_1 - 0.2\, I_2)$$

$$V_2 = I_1 + 5.6\, I_2$$

$\therefore$ $Z_{21} = 1\Omega$ $\qquad$ $Z_2 = 5.6\ \Omega$

$$\Delta z = Z_{11} Z_{22} - Z_{12} Z_{21} = 12.5 \times 5.6 - 1 \times 1$$

$$= 69$$

Converting Z to h parameters

$$\begin{bmatrix} h''_{11} & h''_{12} \\ h''_{21} & h''_{22} \end{bmatrix} = \begin{bmatrix} 12.32 & 0.178 \\ -0.178 & 0.178 \end{bmatrix}$$

h – parameter of the interconnected network is given by,

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} h'_{11} + h''_{11} & h'_{12} + h''_{12} \\ h'_{21} + h''_{21} & h'_{22} + h''_{22} \end{bmatrix}$$

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} 44.98 & 0.511 \\ -3.011 & 0.261 \end{bmatrix}$$

Q.6(a) The characteristic impedance of a high frequency line is 100 Ω. If its terminated by a load impedance of 100 + j100Ω. Using smith chart, find out : (i) VSWR, (ii) Reflection coefficient, (iii) Impedance at $(1/10)^{th}$ of wave length away from load, (iv) VSWR minimum and VSWR maximum away from the load. **[10]**

(A) $Z_L = 100 + j100$

$$\overline{Z_L} = \frac{100 + j100}{100} = 1 + j1$$

i) VSWR = 2.6

ii) Reflection coefficient = $0.44\angle 64^\circ$

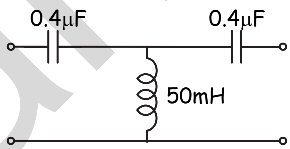
iii) Impedance at $\frac{1}{10}^{th}$ of λ away from load = 2.5 − j0.4

iv) VSWR minimum = 0.25 λ + 0.088 = 0.338λ

VSWR maximum = 0.25 λ − 0.162 λ = 0.088 λ

(Smith chart is attached behind)

Q.6(b) Find the characteristic impedances, cut-off frequency and passband frequency for given network. **[5]**



(A) The network is a high pass filter

2C = 0.4 μF L = 50 mH C = 0.2 μF

(i) Characteristic impedance

$$K = \sqrt{\frac{L}{C}} = \sqrt{\frac{50 \times 10^{-3}}{0.2 \times 10^{-6}}} = 500\ \Omega$$

(ii) Cut-off frequency

$$f_c = \frac{1}{4\pi\sqrt{LC}} = \frac{1}{4\pi\sqrt{50 \times 10^{-3} \times 0.2 \times 10^{-6}}} = 795.77\ \text{Hz}$$

(iii) Pass band : The pass band is from 795.77 Hz to infinite frequency.

Q.6(c) Explain various types of filters. **[5]**

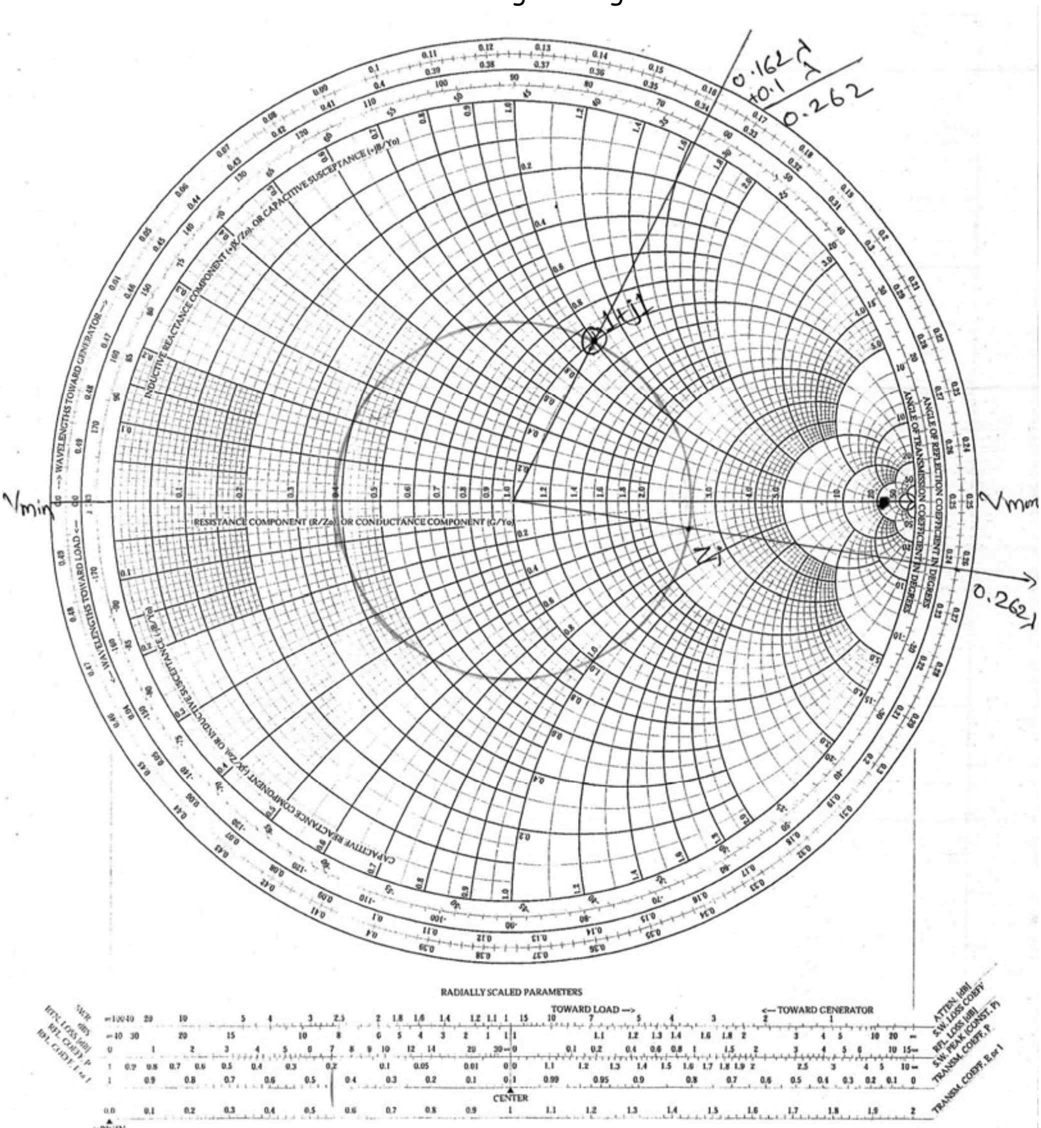
(A) Filters are classified into four categories :

1) *Low Pass Filter* : A low-pass filter allows all frequencies up to a certain cut-off frequency to pass through it and attenuates all the other frequencies above the cut-off frequency.
2) *High Pass Filter* : A high pass filter attenuates all the frequency below the cut-off frequency and allows all other frequencies above the cut-off frequency to pass through it.

3) *Band Pass Filter* : A band pass filter allows a limited band of frequencies to pass through it and attenuates all other frequencies below or above the frequency band.
4) *Band Stop Filter* : A band stop filter attenuates a limited band of frequencies but allows all other frequencies to pass through it.

The complete Smith Chart

Black Magic Design



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