Q.1(a) Test whether $P(s) = s^5 + 12s^4 + 45s^3 + 60s^2 + 44s + 48$ is Hurwitz polynomial. **[5]**

(A) $P(s) = s^5 + 12s^4 + 45s^3 + 60s^2 + 44s + 48$

s^5	1	45	44
s^4	12	60	48
s^3	40	40	0
s^2	48	48	
s^1	0	0	

As complete row becomes zero we will form an auxiliary equation.

$A(s) = 48s^2 + 48$

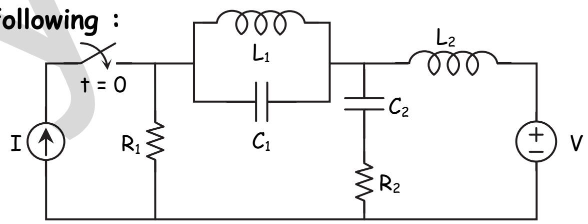
$A'(s) = 96s + 0$

Replacing zeroth row with $A'(s)$

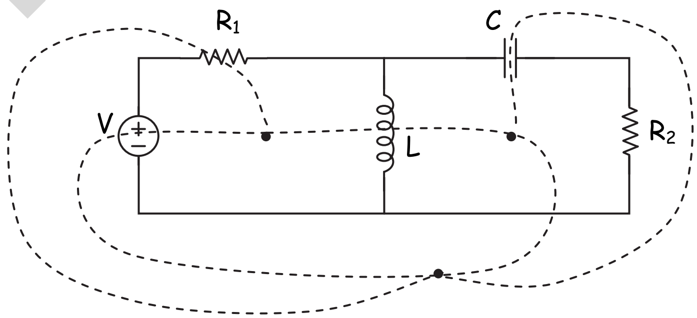
s^5	1	45	44
s^4	12	60	48
s^3	40	40	0
s^2	48	48	
s^1	96	0	
s^0	48		

As all the elements in the 1st columns are positive given f^n is a Hurwitz polynomial.

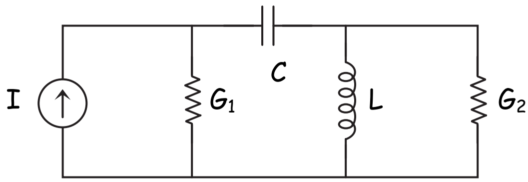
Q.1(b) Draw dual of the following : **[5]**



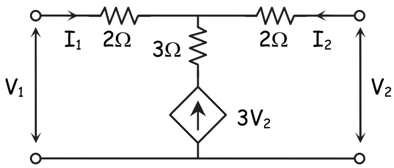
(A)



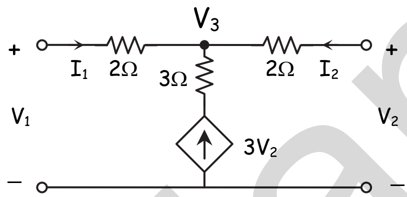
So dual network is given by



Q.1(c) Determine the short circuit admittance parameters of the network shown : **[5]**



(A)



Apply RCL at V_3

$$\frac{V_3 - V_1}{2} + \frac{V_3 - V_2}{2} = 3V_2$$

$$V_3\left(\frac{1}{2} + \frac{1}{2}\right) = 3V_2 + \frac{V_2}{2} + \frac{V_1}{2}$$

$$V_3 = 0.5\,V_1 + 3.5\,V_2 \qquad \text{...(1)}$$

Apply KCL at V_1

$$I_1 = \frac{V_1 - V_3}{2}$$

$$I_1 = \frac{V_1}{2} = \frac{1}{2}\left(0.5V_1 + 3.5\,V_2\right)$$

$$I_1 = V_1(0.25) - V_2(1.75)$$

Comparing with $I_1 = y_{11}V_1 + y_{12}\,V_2$

$y_{11} = 0.25\,\mho \qquad y_{12} = -1.75\,\mho$

Apply KCL at V_2

$$I_2 = \frac{V_2 - V_3}{2}$$

$$I_2 = \frac{V_2}{2} - \frac{1}{2}\left(0.5\,V_1 + 3.5\,V_2\right)$$

$$I_2 = -0.25\,V_1 - 1.25\,V_2$$

$y_{21} = -0.25\,\mho\,, \qquad y_{22} = -1.25\,\mho$

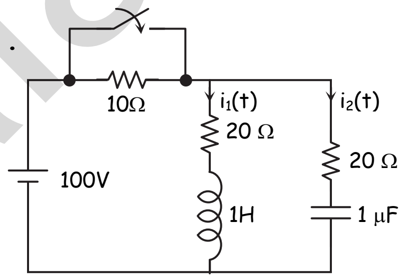
Q.1(d) State and prove final value theorem of Laplace transform. **[5]**

(A) If L {f(t)} = F(s) then $\lim\limits_{t\to 0} f(t) = \lim\limits_{s\to 0} sF(s)$

Proof : We know that

$$L\{f'(t)\} = s\,F(s) - f(0)$$
$$s\,F(s) = L\{f'(t)\} + f(0)$$
$$s\,F(s) = \lim_{s\to 0}\int_0^\infty f'(t)e^{-st}\,dt + f(0)$$
$$= \int_0^\infty \lim_{s\to 0}[f'(t)e^{-st}]\,dt + f(0)$$
$$= \int_0^\infty f'(t)\,dt + f(0)$$
$$= |f(t)|_0^\infty + f(0)$$
$$= \lim_{t\to\infty} f(t) - f(0) + f(0)$$
$$= \lim_{t\to\infty} f(t)$$

Q.2(a) The network shown in figure, a steady state is reached with the switch open. At t = 0, the switch is closed. Determine $V_c(0^-)$, $i_1(0^+)$, $i_2(0^+)$, $\frac{di_1}{dt}(0^+)$ and $\frac{di_2}{dt}(0^+)$. **[10]**



(A) **Step 1:** At $t = 0^-$, switch is open

$i_2(0^-) = 0$

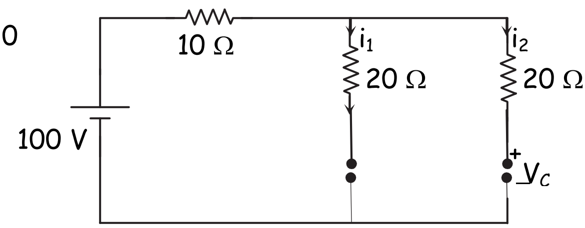
finding $i_1(0^-)$

$100 - 10\,i_1 - 20\,i_1 = 0$

$100 = 30\,i_1$

$\therefore\ i_1 = \frac{100}{30} = \frac{10}{3}A$

$\therefore\ i_1(0^-) = \frac{10}{3}A$

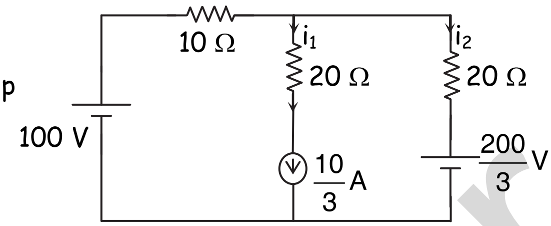


$V_C(0^-) = 20 \times i_1(0^-) = 20\times\frac{10}{3}$

$V_C(0^-) = \frac{200}{3}V$

Step 2: At $t = 0^+$

$i_1(0^+) = \frac{10}{3} A$



Apply KVL to the outer loop

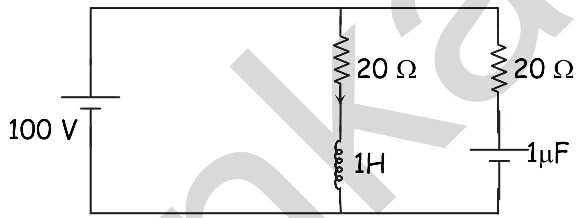
$$100 - 20i_2 - \frac{200}{3} = 0$$

$$20\, i_2 = 100 - \frac{200}{3}$$

$$i_2(0^+) = 1.67\ A$$

Step 3: At $t > 0$

Apply KVL to loop (1)



$$100 - 20\, i_1 - \frac{di_1}{dt} = 0$$

$$\frac{di_1}{dt} = 100 - 20\, i\,(0^+)$$

$$= 100 - \frac{20 \times 10}{3}$$

$$\frac{di_1}{dt} = 33.33\ A/s$$

Apply KVL to outer loop

$$100 - 20\, i_2 - \frac{1}{10^{-6}}\int i_2 dt = 0$$

Differentiate w.r.t. t

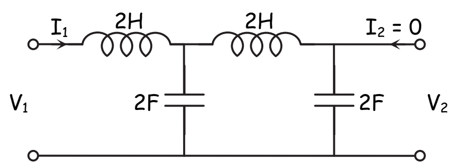
$$0 - 20\frac{di_2}{dt} - 10^6\, i_2 = 0$$

$$-20\frac{di_2(0^+)}{dt} = 10^6\, i_2(0^+)$$

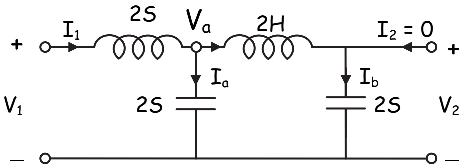
$$\frac{di_2}{dt} = \frac{-10^6(1.67)}{20}$$

$$\frac{di_2}{dt} = -83500\ A/s$$

Q.2(b) Find the network functions $\frac{V_1}{I_1}, \frac{V_2}{I_1}, \frac{V_2}{V_1}$ for the network shown : **[5]**



(A) Let $V_2 = 1$



$I_b = V_2 \times 2S$

$V_a = V_2 + I_b \cdot 2S$

$= 1 + 2S \cdot 2S$

$V_a = 1 + 4S^2$

$I_a = V_a \cdot 2S$

$= (1 + 4S^2)2S$

$I_a = 2S + 8S^3$

$I_1 = I_a + I_b$

$= 2S + 8S^3 + 2S$

$I_1 = 8S^3 + 4S$

$V_1 = V_a + I_i 2S$

$= 1 + 4S^2 + (8S^3 + 4S)2S$

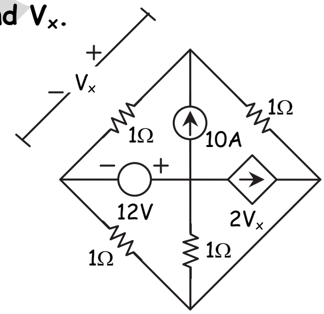
$= 1 + 4S^2 + 16S^4 + 8S^2$

$V_1 = 16S^4 + 12S^2 + 1$

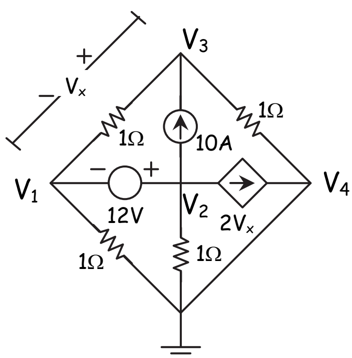
$$\frac{V_1}{I_1} = \frac{16S^4 + 12S^2 + 1}{8S^3 + 4S}$$

$$\frac{V_2}{I_1} = \frac{1}{8S^3 + 4S} \quad \& \quad \frac{V_2}{V_1} = \frac{1}{16S^4 + 12S^2 + 1}$$

Q.2(c) In the circuit shown in figure, find V_x. **[5]**



(A)



$V_4 = 0$ V

$V_x = V_1 - V_3$ ...(1)

Relation between V_1 & V_2

$V_2 - V_1 = 12$

$-V_1 + V_2 + 0V_3 = 12$ ...(2)

Supernode equation

$$\frac{V_1 - V_3}{1} + \frac{V_1}{1} + \frac{V_2}{1} + 10 + 2V_x = 0$$

$V_1 - V_3 + V_1 + V_2 + 10 + 2(V_1 - V_3) = 0$ ...(from 1)

$4V_1 + V_2 - 3V_3 = -10$ ...(3)

KCL to V_3

$$\frac{V_3 - V_1}{1} + \frac{V_3}{1} = 10$$

$-V_1 + 0V_2 + 2V_3 = 10$ ...(4)

Solving (2), (3) & (4)

$V_1 = -2$ V

$V_2 = 10$ V

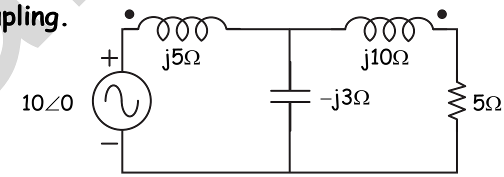
$V_3 = 4$ V

$V_x = V_1 - V_3$

$V_x = -2 - 4$

$V_x = -6$ V

Q.3(a) Find the voltage across 5 Ω resistor in the network shown below. If K = 0.8 is coefficient of coupling. **[8]**

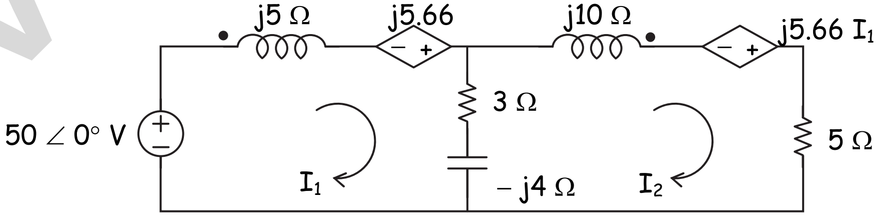


(A) For a magnetically coupled circuit,

$M = K\sqrt{5(10)}$

$X_m = K\sqrt{X_{L_1} X_{L_2}} = 0.8\sqrt{5(10)} = 5.66\ \Omega$

The equivalent circuit in terms of dependent sources can be drawn as



Applying KVL to Mesh 1,

$50 \angle 0° - j5I_1 - 3(I_1 - I_2) + j4(I_1 - I_2) + j5.66I_2 = 0$

$50 \angle 0° = (3 + j1) I_1 - (3 + j1.66) I_2$ … (1)

$(3 + j1) I_2 + (-3 - j1.66) I_2 = 50 \angle 0°$

Applying KVL to Mesh 2,

$j4(I_2 - I_1) - 3(I_2 - I_1) - j10I_2 + j5.66I_1 - 5I_2 = 0$

$j4I_2 - j4I_1 - 3I_2 + 3I_1 - j10I_2 + j5.66I_1 - 5I_2 = 0$

$-j4I_2 + j4I_1 + 3I_2 - 3I_1 + j10I_2 - j5.66I_1 + 5I_2 = 0$

$(-3 - j1.66) I_1 + (8 + j6) I_2 = 0$ … (2)

By Cramer's rule,

$$I_2 = \frac{\begin{vmatrix} 3+j1 & 50\angle 0° \\ -3-j1.66 & 0 \end{vmatrix}}{\begin{vmatrix} 3+j1 & -3-j1.66 \\ -3-j1.66 & 8+j6 \end{vmatrix}} = 8.62 \angle -24.79° \text{ A}$$

$V = 5I_2 = 5(8.62 \angle -24.79°) = 43.1 \angle -24.79°$ V

Q.3(b) Check the positive real function : **[8]**

(i) $F(s) = \dfrac{s^2 + 6s + 5}{s^2 + 9s + 14}$ **(ii)** $F(s) = \dfrac{s^3 + 6s^2 + 7s + 3}{s^2 + 2s + 1}$

(A) (i) $F(s) = \dfrac{s^2 + 6s + 5}{s^2 + 9s + 14}$

Step 1 : Let $F(s) = \dfrac{N(s)}{D(s)} = \dfrac{s^2 + 6s + 5}{s^2 + 9s + 14}$

$$F(s) = \frac{(s+5)(s+1)}{(s+7)(s+2)}$$

The function f(s) has poles at s = –7 & s = –2 & zeros at s = –5 and s = –1

∴ All the poles & zeros are in the left half of s–plane.

Step 2 : As there is no pole on jw axis, the residue test is not carried out.

Step 3 : Even part of N(s) = $m_1 = s^2 + 5$

Odd part of N(s) = $n_1 = 6s$

Even part of D(s) = $m_2 = s^2 + 14$

Odd part of D(s) = $n_2 = 9s$

$A(w^2) = m_1m_2 - n_1n_2$

$= (s^2 + 5)(s^2 + 14) - (6s)(9s) \big|_{s = jw}$

$= s^4 - 35s^2 + 70 \big|_{s = jw}$

∴ $A(w^2) = w^4 + 35w^2 + 70$

$A(w^2)$ is positive for all $w \geq 0$

As all the three conditions are satisfied, the function is a positive real function.

(ii) $F(s) = \dfrac{s^3 + 6s^2 + 7s + 3}{s^2 + 2s + 1}$

Step 1 : Let $F(s) = \dfrac{N(s)}{D(s)} = \dfrac{s^3 + 6s^2 + 7s + 3}{s^2 + 2s + 1}$

$$F(s) = \frac{s^3 + 6s^2 + 7s + 3}{(s+1)^2}$$

From above both the poles are lying at $s = -1$

Now we will check for N(s)

$N(s) = s^3 + 6s^2 + 7s + 3$

The Routh array of N(s) is given by

s^3	1	7
s^2	6	3
s^1	$\frac{39}{6}$	0
s^0	3	

As all elements in 1st column are +ve, N(s) is Hurwitz polynomial.

$\therefore$ 1st condition is satisfied.

Step 2 : As there is no pole on jw axis, residue test need not to be carried out.

Step 3 : Even part of N(s) = $m_1 = 6s^2 + 3$

Odd part of N(s) = $n_1 = s^3 + 7s$

Even part of D(s) = $m_2 = s^2 + 1$

Odd part of D(s) = $n_2 = 2s$

$A(w^2) = m_1m_2 - n_1n_2$

$= (6s^2 + 3)(s^2 + 1) - (s^3 + 7s)(2s) \,\big|_{s=jw}$

$= 4s^4 - 5s^2 + 3 \,\big|_{s=jw}$

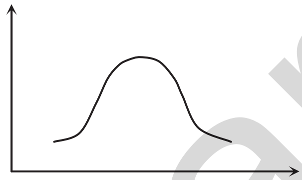
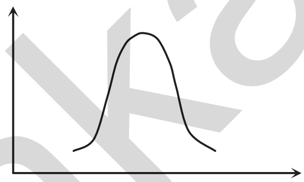
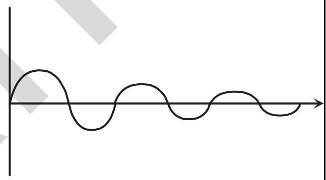
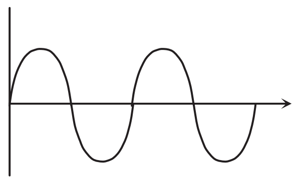
$A(w^2) = 4w^4 + 5w^2 + 3$

$\therefore$ We can say that $A(w^2)$ is positive for all $w \geq 0$

As all three conditions are satisfied, function f(s) is a positive real function.

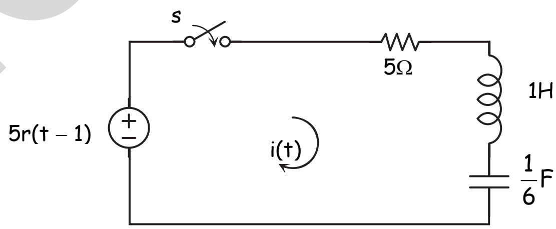
Q.3(c) List the types of damping in series R-L-C circuit and mention the condition for each damping. [4]

(A)

	Nature of Roots	System	Response
1.	Negative Real unequal	Over damped $k_1 e^{p_1 t} + k_2 e^{p_2 t}$	
2.	Negative Real equal	Critically damped $k_1 e^{pt} + k_2 t e^{pt}$	
3.	Complex conjugate	Underdamped $e^{\sigma t}\left[k_1 \cos \omega t + k_2 \sin \omega t\right]$	
4.	Conjugate imaginary	Oscillatory $k_1 \cos \omega t + k_2 \sin \omega t$	

where $\alpha = \dfrac{R}{2L}$ and $\omega_0 = \dfrac{1}{\sqrt{LC}}$

Q.4(a) For the network shown, determine the current i(t) when the switch is closed at t = 0 with zero initial conditions. [8]

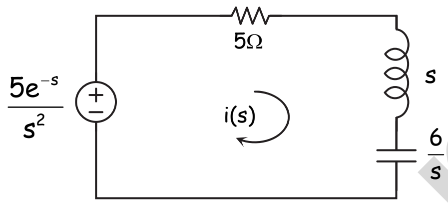


(A) Step 1: At $t = 0^-$, switch is open

$i(0^-) = 0$ A

$V_c(0^-) = 0$V

Step 2 : At t > 0, switch is closed.
Applying KVL



$$5\frac{e^{-s}}{s^2} - 5\,i(s) - s\,i(s) - \frac{6}{s} i(s) = 0$$

$$\frac{5e^{-s}}{s^2} = i(s)\left(5 + s + \frac{6}{s}\right)$$

$$\frac{5e^{-s}}{s^2} = \frac{5s + s^2 + 6}{s}\; i(s)$$

$$(s^2 + 5s + 6)\; i(s) = \frac{5e^{-s}}{s}$$

$$i(s) = \frac{5e^{-s}}{s(s^2 + 5s + 6)}$$

$$i(s) = \frac{5e^{-s}}{s(s+3)(s+2)}$$

Let $\quad i_1(s) = \dfrac{1}{s(s+3)(s+2)} = \dfrac{A}{s} + \dfrac{B}{s+3} + \dfrac{C}{s+2}$

$$1 = A(s + 3)(s + 2) + B\,s(s + 2) + C\,s(s + 3)$$

$$A = \frac{1}{6}, \qquad B = \frac{1}{3}, \qquad C = -\frac{1}{2}$$

$$i(C) = 5e^{-s} \times i_1(s)$$

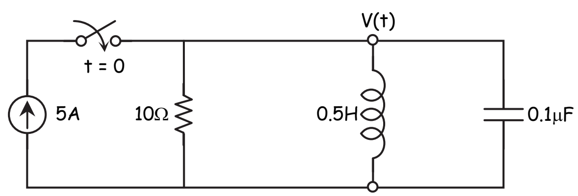
$$= 5e^{-s}\left[\frac{1}{6s} + \frac{1}{3(s+3)} - \frac{1}{2(s+2)}\right]$$

$$i(s) = \frac{5e^{-s}}{6s} + \frac{5\,e^{-s}}{3(s+3)} - \frac{5e^{-s}}{2(s+2)}$$

Taking inverse laplace transform

$$i(t) = \frac{5}{6}u(t-1) + \frac{5}{3}e^{-3(t-1)}\,u(t-1) - \frac{5}{2}e^{-2(t-1)}\,u(t-1)$$

Q.4(b) In the given network switch is closed at t = 0. Solve for V, $\frac{dV}{dt}$, $\frac{d^2V}{dt^2}$ at t = 0^+. **[8]**

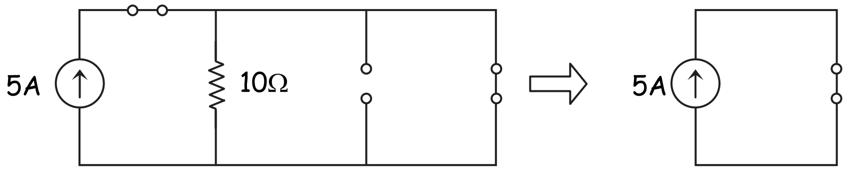


(A) Step 1: At t = 0^-, switch is open.

$i_L(0^-) = 0$ A

$V_c(0^-) = 0V$ & $V(0^-) = 0V$

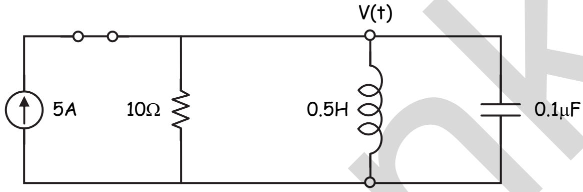
Step 2: At t = 0^+, switch is closed.



$i_L(0^+)$ = 0A

$V_1(0^+)$ = 0V, $V(0^+)$ = 0V

Step 3: At t > 0



Applying KCL to V

$$\frac{V}{10}+\frac{1}{0.5}\int V\,dt+0.1\times10^{-6}\frac{dV}{dt} = 5 \qquad ...(1)$$

At t = 0^+

$$\frac{V(0^+)}{10}+0+0.1\times10^{-6}\frac{dV}{dt}(0^+) = 5$$

$$0.1\times10^{-6}\frac{dV}{dt}(0^+) = 5-0$$

$$\frac{dV}{dt}(0^+) = \frac{5}{0.1\times10^{-6}}$$

$$\frac{dV}{dt}(0^+) = 50\times10^{+6}\text{ v/s}$$

Diff. equation (1) w.r.t. t

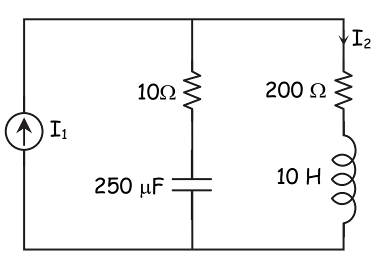
$$\frac{dV}{10\,dt}+\frac{1}{0.5}V(0^+)+0.1\times10^{-6}\frac{d^2V}{dt^2} = 0$$

$$0.1\times10^{-6}\frac{d^2V}{dt^2} = \frac{-dV}{10dt}$$

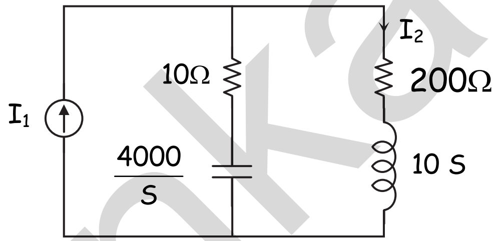
$$\frac{d^2V}{dt^2} = -\frac{50\times10^6}{10\times0.1\times10^{-6}}$$

$$\frac{d^2V}{dt^2} = -50\times10^{12}\text{ V/s}^2$$

Q.4(c) Obtain pole-zero plot for $\frac{I_2}{I_1}$. **[4]**



(A) By C.D.R.



$$I_2 = \frac{\frac{4000}{S}+10}{\frac{4000}{S}+10+200+10S}\times I_1$$

$$\frac{I_2}{I_1} = \frac{\frac{4000+10S}{S}}{\frac{4000+10S+200S+10S^2}{S}}$$

$$\frac{I_2}{I_1} = \frac{4000+10S}{10S^2+210S+4000}$$

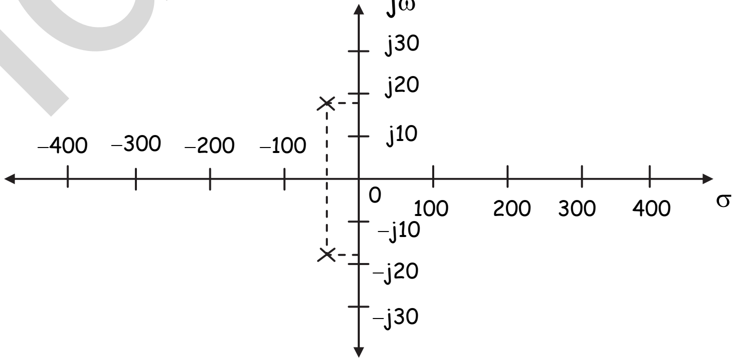
$$\frac{I_2}{I_1} = \frac{10(S+400)}{10\,(S^2+21S+400)}$$

$$\frac{I_2}{I_1} = \frac{S+400}{S^2+21S+400}$$

$Z_1 = -400$

$P_1 = -10.5 + 17.02j$

$P_2 = -10.5 - 17.02j$



Q.5(a) Synthesize the driving point function using Foster-I and Foster-II form : $Z(s) = \dfrac{2(s^2+1)(s^2+9)}{s(s^2+4)}$ **[10]**

(A) Foster – I form :

A degree of numerator is greater than degree of denominator, division is first carried out.

$$Z(s) = \frac{4(s^2++1)(s^2+9)}{s(s^2+4)} \qquad = \frac{4s^4+40s^2+36}{s^3+4s}$$

$$\begin{array}{r|l} & 4s \\ s^3+4s & 4s^4+40s^2+36 \\ & \underline{4s^4+16s^2} \\ & \quad 24s^2+36 \end{array}$$

$$\therefore \; Z(s) = \text{Quotient} + \frac{\text{Remainder}}{\text{Diviser}} = 4s+\frac{24s^2+36}{s^3+4s} = 4s+\frac{24s^2+36}{s(s^2+4)}$$

By partial fraction expansion

$$Z(s) = 4s+\frac{k_0}{s}+\frac{2k_1 s}{s^2+4}$$

$$k_0 = sz(s)\big|_{s=0} = \frac{4(1)(9)}{4} = 9$$

$$k_1 = \left.\frac{(s^2+4)z(s)}{2s}\right|_{s^2=-4} = \frac{4(-4+1)(-4+9)}{2(-4)} = \frac{15}{2}$$

$$Z(s) = 4s+\frac{9}{s}+\frac{15s}{s^2+4}$$

1st term $\Rightarrow 4s \quad \Rightarrow$ inductor of 4H

2nd term $\Rightarrow \dfrac{9}{s} \quad \Rightarrow$ capacitor of $\dfrac{1}{9}$F

3rd term $\Rightarrow$ parallel LC network

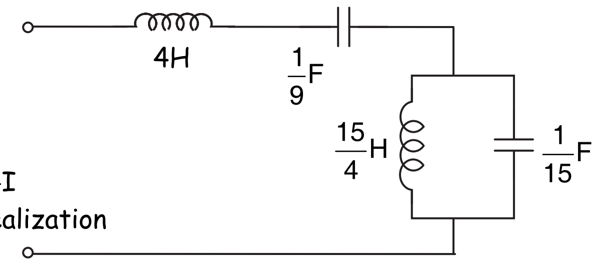
For parallel LC network

$$Z_{LC}(s) = \frac{\left(\frac{1}{c}\right)_s}{s^2+\frac{1}{LC}}$$

Comparing the terms

$$\therefore \; C = \frac{1}{15}\text{F and } L = \frac{15}{4}\text{H}$$

$\therefore$ Corresponding network is as follows.



F–I
Realization

Foster II form :

It is obtain by partial fraction expansion of admittance function

$$Y(s) = \frac{s(s^2+4)}{4(s^2+1)(s^2+9)}$$

By partial fraction expansion

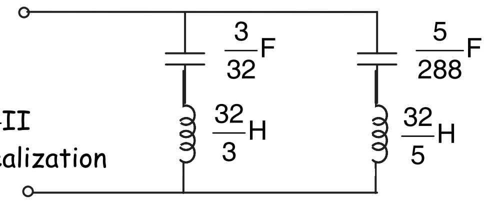
$$Y(s) = \frac{2k_1 s}{s^2+1} + \frac{2k_2 s}{s^2+9}$$

$$k_1 = \frac{(s^2+1)}{2s} y(s)\bigg|_{s^2=-1} = \frac{(-1+4)}{8(-1+9)} = \frac{3}{64}$$

$$k_2 = \frac{s^2+9}{2s} y(s)\bigg|_{s^2=-9} = \frac{(-9+4)}{8(-9+1)} = \frac{5}{64}$$

$$y(s) = \frac{\left(\frac{3}{32}\right)s}{s^2+1} + \frac{\left(\frac{5}{32}\right)s}{s^2+9}$$

Above two terms presents admittance of series LC network for series LC network $\Rightarrow Y_{LC}(s) = \frac{\left(\frac{1}{L}\right)s}{s^2+\frac{1}{LC}}$



F–II
Realization

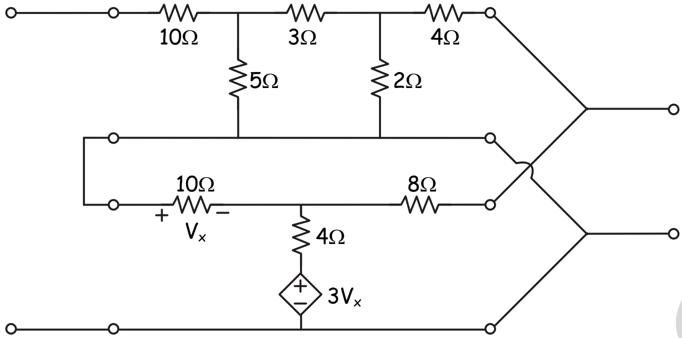
$\therefore$ By comparison

$L_1 = \frac{3L}{3}H \qquad C_1 = \frac{3}{32}F$

$L_2 = \frac{32}{5}H \qquad C_2 = \frac{5}{288}F$

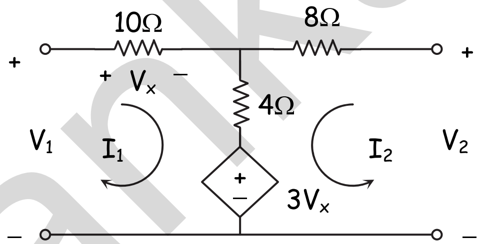
Corresponding network is as shown.

Q.5(b) Obtain hybrid parameter of the inter-connected network. [10]



(A) Separate the two networks

1^{st} network



$V_x = 10 I_1$

Apply KVL to loop of I_1

$V_1 - 10I_1 - 4(I_1 + I_2) - 3V_x = 0$

$V_1 = 10 I_1 + 4(I_1 + I_2) + 3(10I_1)$

$V_1 = 10I_1 + 4I_1 + 4I_2 + 30I_1$

$V_1 = 44I_1 + 4I_2$

Comparing with $V_1 = z_{11} I_1 + z_{12}I_2$

$\therefore\ z_{11} = 44\Omega \qquad z_{12} = 4\Omega$

Apply KVL to loop of I_2

$V_2 - 8I_2 - 4(I_2 + I_1) - 3V_x = 0$

$V_2 = 8I_2 + 4(I_2 + I_1) + 3\ (10I_1)$

$V_2 = 34I_1 + 12I_2$

Comparing with

$V_2 = z_{21}I_1 + z_{22}\ I_2$

$\therefore\ z_{21} = 34\ \Omega \qquad z_{22} = 12\Omega$

$\Delta z = z_{11}\ z_{22} - z_{21}\ z_{12} = 44 \times 12 - 34 \times 4$

$\Delta z = 392$

Now finding h-parameters from Z-parameter

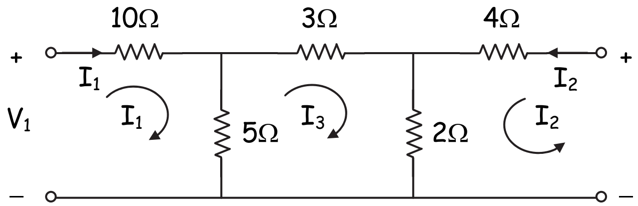
$$h'_{11} = \frac{\Delta z}{z_{22}} = \frac{392}{12} = 32.66$$

$$h''_{12} = \frac{z_{12}}{z_{22}} = \frac{4}{12} = 0.333$$

$$h'_{21} = \frac{-z_{21}}{z_{22}} = \frac{-34}{12} = -2.833$$

$$h'_{22} = \frac{1}{z_{22}} = \frac{1}{12} = 0.0833$$

$$\therefore \begin{bmatrix} h'_{11} & h'_{12} \\ h'_{21} & h'_{22} \end{bmatrix} = \begin{bmatrix} 32.66 & 0.333 \\ -2.833 & 0.0833 \end{bmatrix}$$



Apply KVL to I_3

$$-5(I_3 - I_1) - 3I_3 - 2(I_3 + I_2) = 0$$

$$-10I_3 + 5I_1 - 2I_2 = 0$$

$$I_3 = \frac{-5I_1 + 2I_2}{-10}$$

$$I_3 = 0.5\, I_1 - 0.2\, I_2 \qquad \text{...(1)}$$

Apply KVL to I_1

$$V_1 - 10I_1 - 5(I_1 - I_3) = 0$$

$$V_1 = 10I_1 + 5I_1 - 5(0.5I_1 - 0.2I_2)$$

$$V_1 = 12.5I_1 + I_2$$

$\therefore \; Z_{11} = 12.5\Omega$ & $Z_{12} = 1\Omega$

Apply KVL to I_2

$$V_2 - 4I_2 - 2(I_2 + I_3) = 0$$

$$V_2 = 4I_2 + 2I_2 + 2(0.5I_1 - 0.2\, I_2)$$

$$V_2 = I_1 + 5.6\, I_2$$

$\therefore \; Z_{21} = 1\Omega \qquad Z_2 = 5.6\ \Omega$

$\Delta z = Z_{11} Z_{22} - Z_{12} Z_{21} \qquad = 12.5 \times 5.6 - 1 \times 1$

$= 69$

Converting Z to h parameters

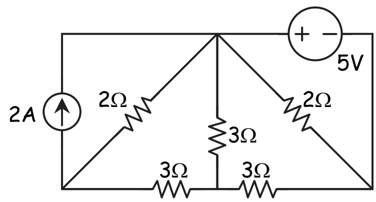
$$\begin{bmatrix} h''_{11} & h''_{12} \\ h''_{21} & h''_{22} \end{bmatrix} = \begin{bmatrix} 12.32 & 0.178 \\ -0.178 & 0.178 \end{bmatrix}$$

h – parameter of the interconnected network is given by,

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} h'_{11} + h''_{11} & h'_{12} + h''_{12} \\ h'_{21} + h''_{21} & h'_{22} + h''_{22} \end{bmatrix}$$

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} 44.98 & 0.511 \\ -3.011 & 0.261 \end{bmatrix}$$

Q.6(a) For the network shown below, draw a graph of network. Select a tree and obtain : [10]
(i) Reduced incidence matrix
(ii) f-cut set matrix
(iii) f-tie set matrix



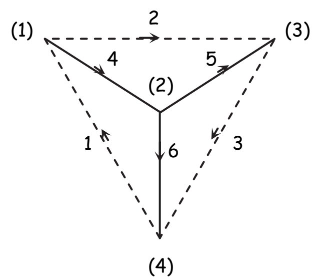
(A) (i) Incidence matrix

No. of Nodes	No. of Branches 1	2	3	4	5	6
1	–1	1	0	1	0	0
2	0	0	0	–1	1	1
3	0	–1	1	0	–1	0
4	1	0	–1	0	0	–1

So the incidence matrix is

$$A_a = \begin{bmatrix} -1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 1 \\ 0 & -1 & 1 & 0 & -1 & 0 \\ 1 & 0 & -1 & 0 & 0 & -1 \end{bmatrix}$$

(ii) f–tieset matrix
Selecting a tree



No. of f-cutset	No. of Branches 1	2	3	4	5	6
1	1	0	0	1	0	1
2	0	1	0	–1	–1	0
3	0	0	1	0	1	–1

∴ The f–tieset matrix is

$$B = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 1 & -1 \end{bmatrix}$$

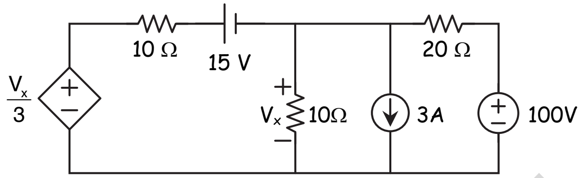
(iii) f–cutset matrix

f-cutset	No. of Branches 1	2	3	4	5	6
4	–1	1	0	1	0	0
5	0	1	–1	0	1	0
6	–1	0	1	0	0	1

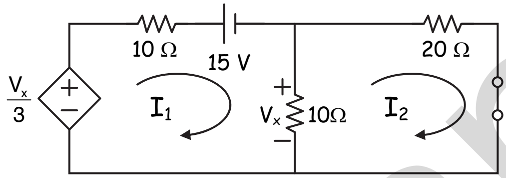
So the f–cutset matrix is

$$Q = \begin{bmatrix} -1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Q.6(b) Find V_x using superposition theorem. **[10]**



(A) Step (1) 15V is active



Apply KVL to I_1

$$\frac{Vx}{3} - 10\,I_1 - 15 - 10(I_1 - I_2) = 0$$

$$V_x = 10(I_1 - I_2)$$

$$\frac{10(I_1 - I_2)}{3} - 10I_1 - 15 - 10(I_1 - I_2) = 0$$

$$I_1\left(\frac{10}{3} - 20\right) + I_2\left(-\frac{10}{3} + 10\right) = 15 \qquad ...(1)$$

Apply KVL to I_2

$$-10(I_2 - I_1) - 20I_2 = 0$$

$$10I_1 - 30I_2 = 0 \qquad ...(2)$$

Solving (1) & (2)

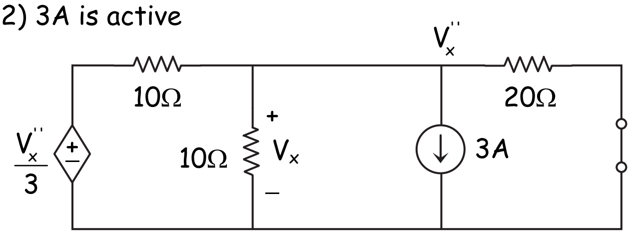
$I_1 = -1.038, \quad I_2 = -0.346$

$V_x = 10(I_1 - I_2)$

$= 10(-1.038+0.346)$

$V_x = -6.92$ V

Step (2) 3A is active



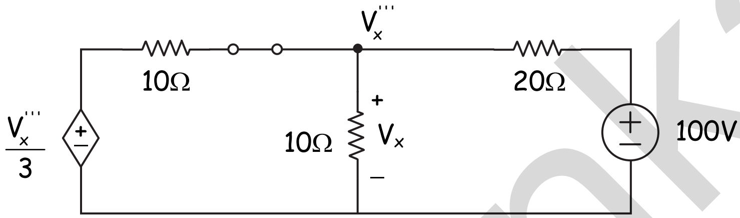
Apply KCL to V_x''

$$\frac{V_x'' - \frac{V_x''}{3}}{10} + \frac{Vx''}{10} + 3 + \frac{V_x''}{20} = 0$$

$$V_x''\left(\frac{1}{10} - \frac{1}{30} + \frac{1}{10} + \frac{1}{20}\right) = -3$$

$$V_x'' = -13.846 \text{ V}$$

Step (3) 100V is active



$$\frac{V_x''' - \frac{V_x'''}{3}}{10} + \frac{V_x'''}{10} + \frac{V_x''' - 100}{20} = 0$$

$$V_x'''\left(\frac{1}{10} - \frac{1}{30} + \frac{1}{10} + \frac{1}{20}\right) = 5$$

$$V_x''' = 23.07 \text{ V}$$

Step (4) finding V_x

$$V_x = V_x' + V_x'' + V_x'''$$

$$V_x = -6.92 - 13.846 + 23.07$$

$$V_x = 2.304 \text{ V}$$

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