

S.E. Sem. III [INST]
Applied Mathematics III
Prelim Question Paper

Time : 3 Hrs.]

[Marks : 80

- N.B.:** (1) Question No. 1 is **compulsory**.
(2) Attempt any **THREE** of the remaining.
(3) Figures to the right indicate full marks.

1. (a) Find Laplace Transformation of $\frac{\sin(3t)}{t}$ [5]
(b) Prove that $f(z) = \cosh z$ is analytic and find its derivate. [5]
(c) Find Fourier series for $f(x) = 16 - x^2$ over $(-4, +4)$ [5]
(d) Prove that $J_{\frac{3}{2}}(x) = \sqrt{\frac{2}{\pi x}} \left(\frac{\sin(x)}{x} - \cos(x) \right)$ [5]
2. (a) Show that the vector field $\bar{F} = (3x^2 y)\mathbf{i} + (x^3 - 2yz^2)\mathbf{j} + (3z^2 - 2y^2z)\mathbf{k}$ is conservative and find $\phi(x, y, z)$ such that $\bar{F} = \nabla\phi$. Also evaluate the line integral $\int \bar{F} \cdot d\bar{r}$ from $(2, 1, 1)$ to $(2, 0, 1)$ [6]
(b) Find the Fourier series for $f(x) = \frac{x-\pi}{4}$; $0 \leq x \leq 2\pi$. Hence prove that [6]
$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots = \frac{\pi}{4}$$

(c) Find Inverse Laplace transform of [8]
i) $\frac{s+19}{(s+9)(s^2+4)}$, ii) $\frac{e^{-3s}}{(s^2+10s+29)}$.
3. (a) Find the analytic function $f(z) = u + iv$ if $u + v = \frac{2\sin(2x)}{e^{2y} + e^{-2y} - 2\cos(2x)}$ [6]
(b) Show that Bessel's Fourier series in $J_2(\lambda_n x)$ for $f(x) = x^2$, $(0 < x < a)$ [6]
Where λ_n positive root of $J_2(ax) = 0$ is $x^2 = 2a \sum_{n=1}^{\infty} \frac{J_2(\lambda_n x)}{\lambda_n J_3(\lambda_n a)}$.
(c) Solve the differential equation $\frac{d^2 y}{dt^2} + 4y = f(t)$, with $y(0) = 0$ & [8]
 $y'(0) = 1$ and $f(t) = \begin{cases} 1 & 0 < t < 1 \\ 0 & 1 < t \end{cases}$.

4. (a) Find the orthogonal Trajectory of $3x^2 - 2x^2y + y^2 = \text{cont.}$ [6]
 (b) Using Greens theorem [6]
 Evaluate $\oint_C ((2xy)dx - (y^2)dy)$ where C is boundary of the region bounded by.
 (c) Find the Fourier integral representation of $f(x) = \begin{cases} 1-x^2 & \text{for } |x| \leq 1 \\ 0 & \text{for } |x| > 1 \end{cases}$. [8]

Hence prove that
$$\int_{\lambda=0}^{\infty} \frac{\cos\left(\frac{\lambda}{2}\right)(\sin \lambda - \lambda \cos \lambda)}{\lambda^3} d\lambda = \frac{3\pi}{16}.$$

5. (a) Find inverse Laplace Transform of $\frac{s}{s^4 + 8s^2 + 16}$ using Convolution [6]
 theorem.
 (b) Find the Bilinear Transform which transform the points $z = 2, i, -2$ of z -plane into the points $w = 1, i, -1$ of the w -plane respectively. Also find fixed points of this transformation. [6]
 (c) Find $\iint_s [(\nabla \times \vec{F}) \cdot \hat{n}] ds$ [8]
 $\vec{F} = (2x - y + z)\mathbf{i} + (x + y - z^2)\mathbf{j} + (3x - 2y + 4z)\mathbf{k}$ over the surface of the cylinder $x^2 + y^2 = 4$ bounded by $z = 9$ and open at the end $z = 0$.
 6. (a) Find the directional derivative of $\phi = xy^2 + yz^2$ at $(2, -1, 1)$ along the [6]
 line $2(x - 2) = y + 1 = z - 1$.
 (b) Obtain the complex form of Fourier series for the function $f(x) = e^{4x}$ [6]
 in $0 < x < 4$.
 (c) Find half range sine series of the function $f(x) = x(3 - x)$ in $0 \leq x \leq 3$ [8]
 Hence prove that :
 (i) $\frac{1}{1^6} + \frac{1}{3^6} + \frac{1}{5^6} + \frac{1}{7^6} + \dots = \frac{\pi^6}{960}$
 (ii) $\sum_{n=1}^{\infty} \frac{1}{(n)^6} = \frac{\pi^6}{945}$.

