

Prelim Paper

Time: 3 Hrs.]

Applied Mathematics - III

[Marks : 80

- N.B.:** (1) Question No. 1 is compulsory.
 (2) Attempt any **THREE** of the remaining.

1. (a) Find Laplace transform of $te^{3t} \cos t$. [5]

(b) Evaluate $\oint_c \frac{z-1}{z^2+2z+5} dz$, where c is $|z+1+i|=2$. [5]

(c) Show that $f(z) = \sinh z$ is analytic. Hence find its derivative. [5]

(d) Compute spearman's rank correlation for the data : [5]

X :	18	20	34	52	12
Y :	39	23	35	18	46

2. (a) Show that the function $w = \frac{4}{z}$ transforms the straight line $x = c$ in the z -plane into circle in w -plane. Find its centre and radius. [6]

(b) Show that $\int_0^\infty e^{-t} \int_0^t \frac{\sin u}{u} du dt = \frac{\pi}{4}$ [6]

(c) Obtain fourier series for $f(x) = \begin{cases} 1 + \frac{2x}{\pi} & -\pi \leq x \leq 0 \\ 1 - \frac{2x}{\pi} & 0 \leq x \leq \pi \end{cases}$ [8]

Hence deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$

3. (a) Evaluate $\oint_c \frac{e^{kz}}{z} dz$, where $c : |z| = 1$. Hence deduce that $\int_0^\pi e^{k \sin \theta} \cos(k \sin \theta) d\theta = \pi$ [6]

(b) For the lines of regression $6y - 5x = 90$, $15x - 8y = 130$ and $\sigma_x^2 = 16$
 Find (i) \bar{x} , \bar{y} (ii) r (iii) σ_y [6]

(c) Solve the differential equation $\frac{dy}{dx} + 2y + \int_0^t y dt = \sin t$ using Laplace transform [8]
 give $y(0) = 1$

4. (a) Find Laurentz's series for $f(z) = \frac{2}{(z-1)(z-2)}$ indicating region of convergence. [6]

(b) Show that $\cos x = \frac{8}{\pi} \sum_{m=1}^\infty \frac{m}{4m^2-1} \sin(2mx)$ If $0 < x < \pi$ [6]

(c) Find Bilinear transformation which maps the points $1, i, -1$ onto the points $i, 0, -i$. Hence find fixed points and image of $|z| < 1$. [8]

5. (a) Solve using Bender-Schmidt method $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$, subject to the conditions [6]

$$u(0, t) = 0, u(1, t) = 0, u(x, 0) = \sin \pi x, 0 \leq x \leq 1$$

- (b) Determine the solution of one-dimensional heat equation under the boundary [6]
conditions, $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$

$$u(0, t) = 0, u(\ell, t) = 0, u(x, 0) = x, (0 < x < \ell), \ell \text{ being length of the rod.}$$

- (c) Find inverse Laplace Transform of [8]

(i) $\log\left(1 + \frac{\alpha^2}{s^2}\right)$ (ii) $\frac{e^{-s}}{s^2 + s + 1}$

6. (a) Obtain complex form of fourier series for $F(x) = e^{ax}$, in $(-\pi, \pi)$ where a is not an [6]
integer.

- (b) Fit a curve $y = a \cdot b^x$ to the following data, using method of least squares. [6]

X :	2	3	4	5	6
Y :	144	172.8	207.4	248.8	298.5

- (c) (i) Evaluate $\int_0^{2\pi} \frac{d\theta}{5 + 3 \sin \theta}$ using Residue Theorem. [4]

- (ii) Evaluate $\int_{-\infty}^{\infty} \frac{dx}{x^2 + 1}$ using Residue Theorem. [4]

