Prelim Paper **Discrete Mathematics**

Time: 3 Hrs.] **[Marks : 80**

N.B.:
(1) Question No. **1** is compulsory.
(2) Solve any THREE out of remaining questions.
(3) Draw neat and clean diagrams.
(4) Assume suitable data if required.

1. (a) Define the following : **[5]**
- (a) Disjoint set
- (b) Partial or division relation
- (c) Symmetric difference
- (d) Antisymmetric relation
- (e) Cartesian product

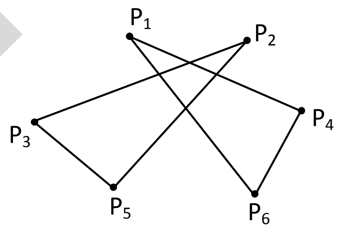
(b) Let A = {1, 2, 3, 4, 6} and let R be the relation on A defined by "x divides y" written x/y. (Note x/y if there exists an integer z such that xz = y) **[5]**
- (i) Write R as a set of ordered pairs.
- (ii) Draw its directed graph.
- (iii) Find the inverse relation of R.

(c) Let (A, *) be an algebraic system, where * is a binary operation such that $\forall a, b \in A$, a * b = a. **[5]**
- (i) Show that * is associative.
- (ii) Is * commutative?

(d) Let m be the positive integer greater than 1. **[5]**
Show that the relation R = {(ab) | a =b(mod m)} i.e. aRb if and only if m divides a − b is equivalence relation on the set of integers.

2. (a) Prove that $((A \cup B) \cap \overline{A}) \cup (\overline{B \cap A}) = (\overline{A \cap B})$. **[4]**

(b) Describe formally the graph shown in figure. **[4]**



(c) Consider the partial order of divisibility on set A. **[6]**
Draw Hass diagram of the poset and determine which poset are linearly ordered.
- (i) A = {1, 2, 3, 5, 6, 10, 15, 30}
- (ii) A = {2, 4, 8, 16, 32}
- (iii) A = {3, 6, 12, 36, 72
- (iv) A = {1, 2, 3, 4, 5, 6, 10, 12, 15, 30, 60}

(d) Let $H = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ **[6]**

be a parity check matrix. Decode the following word related to maximum likelihood technique (Decoding function) associated with e_H. Decode the following : (i) 10100 (ii) 01101 (iii) 11011

3. (a) Define universal and Existential quantifiers. Transcribe the following into logical notation Let the universe of discourse be the real numbers: **[6]**

(i) There are positive values of x and y such that x, y $>$ 0

(ii) For every value of x there is some value of y such that x, y = 1

(iii) There is a value of x such that if y is positive then x + y is negative

(b) Let f : R $\rightarrow$ R be a function defined by f (x) = 2x $-$ 3. **[4]**
Prove that it is bijective hence find inverse.

(c) A bag contains six white marbles and five red marbles. Find the number of ways four marbles can be drawn from the bag if (a) they can be any color; (b) two must be white and two red; (c) they must all be of the same color. **[4]**

(d) Prove that a set of non–zero real numbers forms an abelian group with respect to binary operation '$*$' where '$*$' is defined as **[6]**

$\forall$ a, b $\in$ R

$$a * b = \frac{a.b}{2}$$

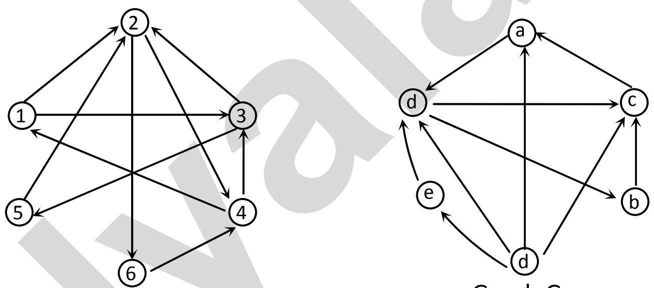
4. (a) State the converse, inverse and contrapositive of the following : **[4]**

(i) If it is cold then he wears a hat

(ii) If an integer is a multiple of 2, then it is even.

(b) Consider the relation R on set of integers defined as xRy iff y = x^k; k is positive integer. Show that R is a partial order relation. **[4]**

(c) What are isomorphic graphs? Show that following two graphs are isomorphic. **[6]**



Graph G_1 Graph G_2

(d) Out of 250 candidates who failed in an examination, it was revealed that 128 failed in mathematics, 87 in physics and 134 in aggregate. 31 failed in mathematics and in Physics, 54 failed in the aggregate and in mathematics, 30 failed in the aggregate and in physics. Find how many candidates failed. **[6]**

(i) in all the three subjects.

(ii) in mathematics but not in physics.

(iii) in the aggregate but not in mathematics.

(iv) in physics but not in aggregate or in mathematics.

5. (a) f : R $\rightarrow$ R is defined by f (x) = x^3 **[4]**

g : R $\rightarrow$ R is defined by g (x) = $4x^2 + 1$

h : R $\rightarrow$ R is defined by h (x) = 7x $-$ 2

Find the rule defining : (i) fog, (ii) gof, (iii) (goh)of, (iv) go(hof)

(b) Show that in a group, $\forall$ a, b $\in$ G, $(a * b)^2 = a^2 * b^2$, iff (G, *) must be abelian. **[4]**

(c) Explain Warshwall's algorithm Let A = {1, 2, 3, 4, 5} and let R be a relation on A Such that R = {(1, 1), (1, 4), (2, 2), (3, 4), (3, 5), (4, 1), (5, 2), (5, 5)} Find transitive closure of R by Warshwall's algorithm. **[6]**

(d) Find the solution of recurrence relation : $a_r + 5a_{r-1} + 6a_{r-2} = 3r^2$ **[6]**

6. (a) $f : R - \left\{\frac{2}{5}\right\} \rightarrow R - \left\{\frac{4}{5}\right\}$ defined by $f(x) = \frac{4x+3}{5x-2}$ show that the function is bijective and find rule for f^{-1}. **[4]**

(b) How many friends must have to guarantee that atleast five of them will have birthday in the same month? **[6]**

(c) A connected planar graph has 9 vertices having degrees 2,2,2,3,3,3,4,4 and 5. How many edges are there ? **[4]**

(d) Let Z^+ is a set of positive integers and a relation R defined on Z^+ by a R b iff a | b then prove that R is a partial order relation and (Z^+ , |) is a poset. **[6]**

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