Prelim Paper **Applied Mathematics - III**

Time: 3 Hrs.] **[Marks : 80**

N.B.: (1) Question No. **1** is **compulsory**.
(2) Attempt any **THREE** of the remaining.

1. (a) Find Laplace transform of $te^{3t}\cos t$. **[5]**

(b) Find $z\{2^k \cdot k^2\}$, $k \geq 0$. **[5]**

(c) Show that $f(z) = \sinh z$ is analytic. Hence find its derivative. **[5]**

(d) Compute spearman's rank correlation for the data : **[5]**

X :	18	20	34	52	12
Y :	39	23	35	18	46

2. (a) Show that the function $w = \frac{4}{z}$ transforms the straight line $x = c$ in the z-plane into circle is w-plane. Find its centre and radius. **[6]**

(b) Show that $\int_0^\infty e^{-t} \int_0^t \frac{\sin u}{u} du dt = \frac{\pi}{4}$ **[6]**

(c) Obtain fourier series for $f(x) = \begin{cases} 1 + \frac{2x}{\pi} & -\pi \leq x \leq 0 \\ 1 - \frac{2x}{\pi} & 0 \leq x \leq \pi \end{cases}$ **[8]**

Hence deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots$

3. (a) Find inverse z-transform of $\frac{z}{z-a}$ in ROC **[6]**

(i) $|z| > a$ (ii) $|z| < a$

(b) For the lines of regression **[6]**

$6y - 5x = 90$, $15x - 8y = 130$ and $\sigma_x^2 = 16$

Find (i) $\bar{x}, \bar{y}$ (ii) r (iii) σy

(c) Solve the differential equation **[8]**

$\frac{dy}{dx} + 2y + \int_0^t y\, dt = \sin t$ using Laplace transform give $y(0) = 1$

4. (a) Find orthogonal trajectory to the family of curves $e^{-x}\cos y + xy = \text{constant}$ in $X - Y$ plane. **[6]**

(b) Show that $\cos x = \frac{8}{\pi} \sum_{m=1}^{\infty} \frac{m}{4m^2 - 1} \sin(2mx)$ **[6]**

If $0 < x < \pi$

(c) Find Bilinear transformation which maps the points 1, i, -1 onto the points i, 0, $-i$. Hence find fixed points and image of $|z| < 1$. **[8]**

5. (a) Find z – transform of $X_k = \begin{cases} 3^k ; k < 0 \\ 2^k ; k \geq 0 \end{cases}$ **[6]**

(b) If $f(x) = c_1 \phi_1(x) + c_2\phi_2(x) + c_3 \phi_3(x)$, where **[6]**
c_1, c_2, c_3 are constant and ϕ_1, ϕ_2, ϕ_3 are orthonormal functions, on the set (a, b).
Show that $\int_a^b [f(x)]^2 \, dx = c_1^2 + c_2^2 + c_3^2$

(c) Find inverse Laplace Transform of **[8]**

(i) $\log\left(1 + \frac{\alpha^2}{s^2}\right)$ (ii) $\frac{e^{-s}}{s^2 + s + 1}$

6. (a) Obtain complex form of fourier series for **[6]**
$F(x) = e^{\alpha x}$, in $(-\pi, \pi)$ where a is not an integer.

(b) Fit a curve $y = a \cdot b^x$ to the following data, using method of least squares. **[6]**

X :	2	3	4	5	6
Y :	144	172.8	207.4	248.8	298.5

(c) Find imaginary part of analytic function whose real part is $e^{2x}(x \cos 2y - y \sin 2y)$, **[8]**
Also verify that v is harmonic function.

❑❑❑❑❑