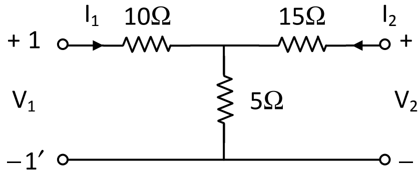
Electrical Network Analysis and Synthesis

Time : 3 Hrs.] Prelim Paper Solution **[Marks : 80**

Q.1(a) Determine the z-parameters or the network shown in the following figure. **[5]**

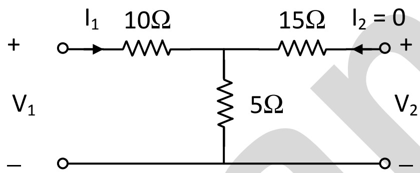


Ans.: Consider z-parameter equations as,

$V_1 = Z_{11} I_1 + Z_{12} I_2$

$V_2 = Z_{21} I_1 + Z_{22} I_2$

Consider Case-I $I_2 = 0$



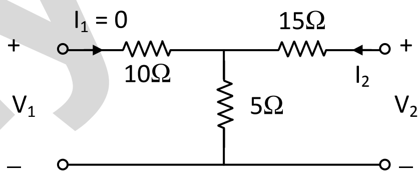
By applying KVL to input side.

$V_1 - 10I_1 - 5I_1 = 0 \quad \because I_2 = 0$

$\therefore V_1 = 15I_1 \qquad \therefore Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = 15\Omega$

Also, $V_2 = 5I_1 \qquad \therefore Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = 5\Omega$

Case-II $I_1 = 0$



By applying KVL to output side.

$V_2 - 15I_2 - 5I_2 = 0 \quad \because I_1 = 0$

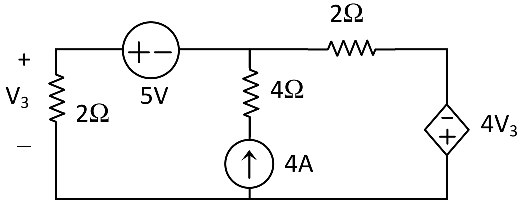
$V_2 = 20I_2 \qquad \therefore Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = 20\Omega$

Also $V_1 = 5I_2 \quad \because I_1 = 0 \qquad \therefore Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = 5\Omega$

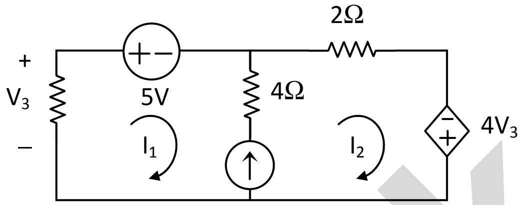
$\therefore$ Z-parameters are,

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 15 & 5 \\ 5 & 20 \end{bmatrix}$$

Q.1(b) By mesh analysis determine the current through 2Ω resistor **[5]**



Ans.:



From above circuit

$I_2 - I_1 = 4$ ...(1)

$V_3 = -2I_1$

By applying KVL to outer loop,

$-2I_1 - 5 - 2I_2 + 4V_3 = 0$

$\therefore \quad -2I_1 - 5 - 2I_2 - 8I_1 = 0$

$-10I_1 - 2I_2 = 5$

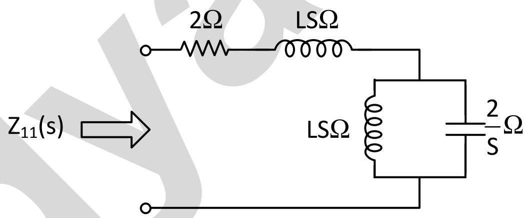
$\therefore \quad 2I_2 + 10I_1 = -5$ ...(2)

By solving equation (1) & (2), we have,

$I_1 = 2.916$ A / $I_2 = -1.083$ A

$\therefore$ Current through 2Ω is -1.083 A

Q.1(c) Determine the driving point impedance function of the one-port network. **[5]**



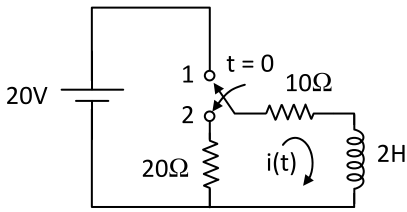
Ans.: $\therefore \quad Z_{11} = (2+S) + \left[S \,||\, \frac{2}{S}\right]$

$$\therefore \quad Z_{11} = 2 + S + \left[\frac{\frac{S.2}{S}}{\frac{S+2}{S}}\right] = 2 + S + \frac{2S}{S^2+2}$$

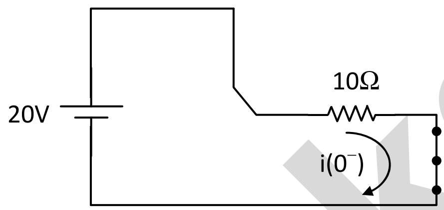
$$= \frac{2s^2 + 4 + s^3 + 2s + 2s}{s^2+2}$$

$$\therefore \quad Z_{11}(s) = \frac{s^3 + 2s^2 + 4s + 4}{s^2+2}$$

Q.1(d) **Find i, $\frac{di}{dt}$ at t = 0t.** **[5]**



Ans.: Step 1: Consider circuit at time $t = 0^-$



$$i(0^-) = \frac{20}{10} = 2A$$

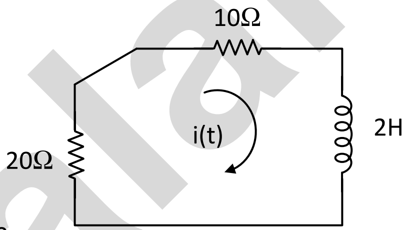
$$i(0^-) = I_0 = 2A$$

At time $t = 0^+$ inductor acts as current source

$\therefore$ $I(0^-) = I(0^+) = I_0 = 2A$

$\therefore$ $I(0^+) = 2A$

Consider circuit at time t > 0.



By applying KVL to loop.

$$-20i(t) - 10i(t) - 2\frac{di(t)}{dt} = 0$$

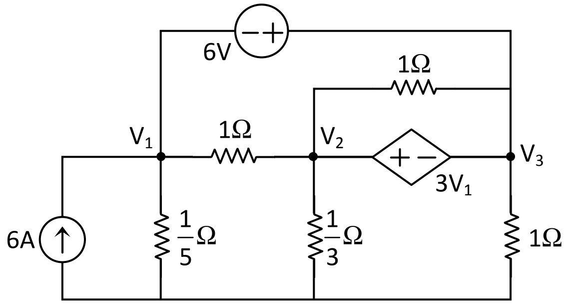
$$\frac{2di(t)}{dt} = -30i(t)$$

put $t = 0^+$

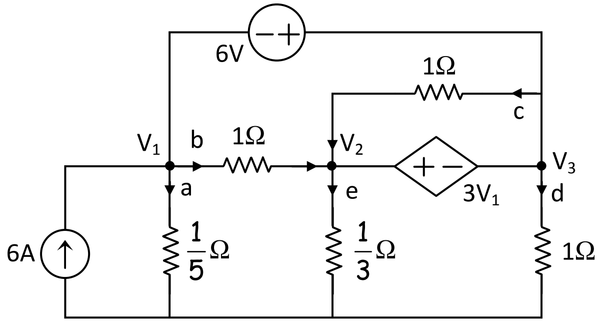
$$\therefore \frac{di(0^+)}{dt} = \frac{-30}{2} i(0^+) = -30 \text{ A/S}$$

$$\therefore \frac{di(0+)}{dt} = 30 \text{ A/s}$$

Q.2(a) **Find nodal voltages for the given circuit.** **[10]**



Ans.:



By observing the above circuit,

$V_1 - V_3 = -6$ ...(1)

Also $V_2 - V_3 = 3V_1$

$\therefore \quad 3V_1 - V_2 + V_3 = 0$ ...(2)

By applying KCL at nodes with supernodes,

$6 - a - b - c - d + c + b - e = 0$

$$6 - \frac{V_1}{\frac{1}{5}} - \frac{V_3}{1} - \frac{V_2}{\frac{1}{3}} = 0$$

$-5V_1 - 3V_2 - V_3 = -6$

$\therefore \quad 5V_1 + 3V_2 + V_3 = 6$ ...(3)

By solving equations (1), (2) & (3) we have,

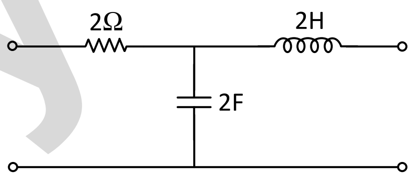
$V_1 = -1V, V_2 = 2V, V_3 = 5V$

$\therefore \quad V_1 = -1\ V$

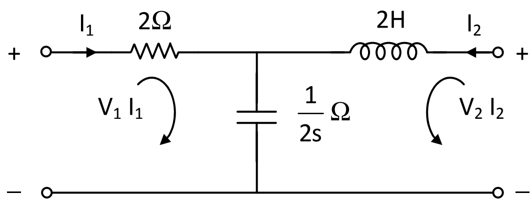
$V_2 = 2V$

$V_3 = 5V$

Q.2(b) Determine transmission parameter or the following network. [5]



Ans.: The Laplace transformed network is given as,



We ABCD parameter equations are

$V_1 = Av_2 - BI_2$ ...(1)

$I_1 = CV_2 - DI_2$ ...(2)

By applying KVL to loop (1) we have,

$$V_1 - 2I_1 - \frac{1}{2S}(I_1 + I_2) = 0$$

$$V_1 = \left(2 + \frac{1}{2S}\right)I_1 + \frac{1}{2S}I_2 \quad \text{...(3)}$$

By applying KVL to loop (2) we have,

$$V_2 - 2SI_2 - \frac{1}{2S}(I_1 + I_2) = 0$$

$$V_2 = \frac{1}{2S}I_1 + \left(2S + \frac{1}{2S}\right)I_2 \quad \text{...(4)}$$

$$\therefore \quad I_1 = 2SV_2 - (4S^2 + 1)I_2 \quad \text{...(5)}$$

Consider equation (3) and put (5) in (3)

$$\therefore \quad V_1 = \frac{(4S+1)}{2S}\left[2SV_2 - (4S^2+1)I_2\right] + \frac{1}{2S}I_2$$

$$V_1 = (4S+1)V_2 - \frac{(16S^3 + 4S^2 + 4S + 1)}{2S}I_2 + \frac{1}{2S}I_2$$

$$V_1 = (4S+1)V_2 - (8S^2 + 2S + 2)I_2 \quad \text{...(6)}$$

By comparing equation (6) & (5) with equations (1) + (2)

We have,

$A = (4S + 1)$ $\quad$ $B = 8S^2 + 2S + 2$

$C = 2S$ $\quad$ $D = 4S^2 + 1$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} (4S+1) & (8S^2+2S+2) \\ 2S & (4S^2+1) \end{bmatrix}$$

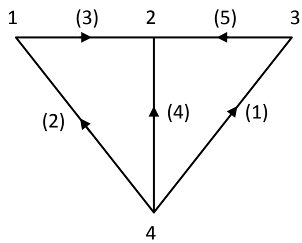
Q.2(c) Draw oriented graph and find number of trees. [5]

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{-1} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{-1} & \mathbf{-1} & \mathbf{-1} \\ \mathbf{-1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix}$$

Ans.: To draw oriented graph complete incidence matrix is required.

$$\therefore \quad [Aa] = \begin{bmatrix} 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 & -1 \\ -1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

For rows as four nodes & five columns as five branches



No. of trees $= \text{Det.}\{[A]\,[A^T]\}$

Consider $\{[A]\,[A^T]\} = \begin{bmatrix} 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 & -1 \\ -1 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & -1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 2 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 2 \end{bmatrix}$

$\therefore$ No. of trees $= \text{Det}\{[A]\,[A^T]\}$

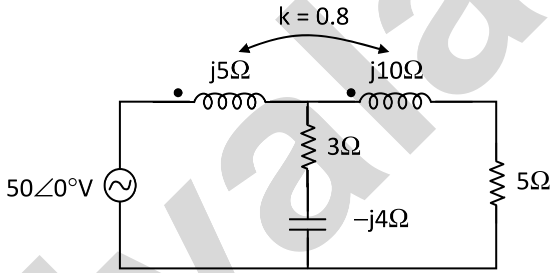
$= \begin{vmatrix} 2 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 2 \end{vmatrix}$

$= 2\,[6 - 1] - (-1)\,[-2-0] + 0\,[1 - 0]$

$= 10 - 2 = 8$

$\therefore$ Number of trees = 8

Q.3(a) Find the voltage across the 5Ω resistor using mesh analysis. **[10]**



Ans.: The equivalent circuit is given as follows :

Let k = 0.8,

$\therefore$ Mutual inductance M is given as

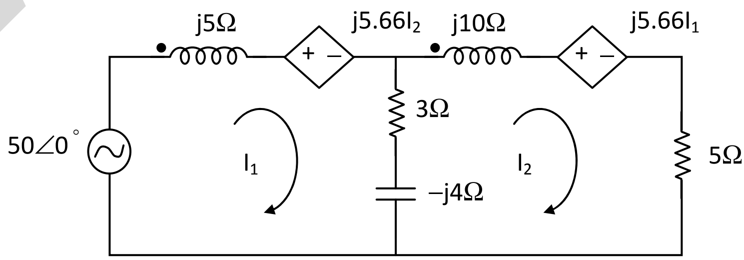
$X_M = k\sqrt{X_{L_1} X_{L_2}}$

$X_M = 0.8\sqrt{5\times 10}$

$X_M = 5.66$

$jX_M = j5.66\ \Omega$

Consider current I_1 & I_2 in Loops (1) & (2) respectively,



By applying KVL to mesh L

$50 - j5I_1 - j5.66I_2 - 3(I_1 - I_2) - (-j4)(I_1 - I_2) = 0$

$(3 + j_1)\, I_1 - (3 - j9.66)\, I_2 = 50$ ...(1)

By applying KVL to Loop (2)

$-j10I_2 - j5.66I_1 - 5I_2 - (-j4)(I_2 - I_1) - 3(I_2 - I_1) = 0$

$(3 - j9.66)\, I_1 - (8 + j6)I_2 = 0$ ...(2)

By solving equation (1) & (2) by using Gramer's Rule,

$$I_2 = \frac{\begin{vmatrix} (3+j) & 50\angle 0^\circ \\ (3-j9.66) & 0 \end{vmatrix}}{\begin{vmatrix} (3+j) & (-3+j9.66) \\ (3-j9.66) & -(8+j6) \end{vmatrix}}$$

$$= \frac{-50\,(3 - j9.66)}{-(3+j)(8+j6) - [(3-j9.66)(-3+j9.66)]}$$

$$= \frac{-150 + j483}{-18 - j26 - 84.315 - j57.96}$$

$$= \frac{-150 + j483}{-102.315 - j83.96}$$

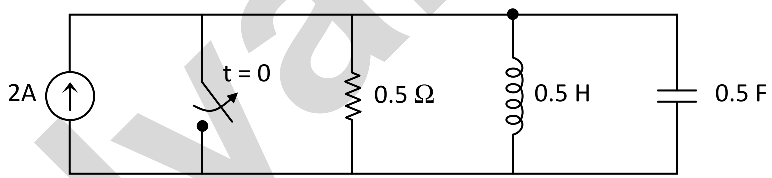
$= -1.438 - j3.54$

$I_2 = 3.82 \angle 67.89$

$V_5\Omega = 5I_2 = 5 \times (3.82 \angle 67.89)$

$V_5\Omega = 19.1\angle 67.89^\circ$

Q.3(b) Determine V(t) for time t > 0 **[10]**



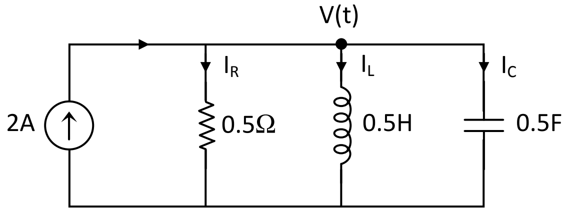
Ans.: To determine v(t) for time t > 0

step-I : circuit at time $t = 0^-$ as switch is closed, therefore current source is short circuited.

$\therefore\ i_L(0^-) = 0$ & $V_C(0^-) = 0$

$\therefore\ V(0^-) = 0$

Step-II : Draw circuit at time t > 0.



By applying KCL at node v(t)

$2 = I_R + I_L + I_C$

$\therefore \quad I_R + I_L + I_C = 2$

$$\frac{V(t)}{0.5} + \frac{1}{0.5}\int_0^t v(t)\,dt + \frac{0.5\,dv(t)}{dt} = 2$$

$$\therefore \quad \frac{0.5\,dv(t)}{dt} + \frac{V(t)}{0.5} + \frac{1}{0.5}\int_0^t v(t)dt = 2 \qquad \text{...(1)}$$

Diff. w.r.t. 't'

$$\frac{0.5\,d^2v(t)}{dt^2} + \frac{1}{0.5}\frac{dv(t)}{dt} + \frac{1}{0.5}v(t) = 0 \qquad \text{...(2)}$$

put $\frac{d}{dt} = s$

$$\therefore \quad 0.5s^2 + \frac{1}{0.5}s + \frac{1}{0.5} = 0$$

$\therefore$ Roots are $s_1, 2 = -2, -2$

i.e. Roots are real, negative and repeated.

$\therefore$ Therefore solution is given as,

$V(t) = k_1\,e^{s_1 t} + k_2 t\,e^{s_2 t}$

$$\therefore \quad V(t) = k_1 e^{-2t} + k_2 t\,e^{-2t} \qquad \text{...(3)}$$

To find k_1 & k_2 put time t = 0

$V(0) = k_1 + k_2(0)$

$\because \quad V(0) = 0$

$\therefore \quad k_1 = 0$

$\therefore$ equation (3) becomes,

$$V(t) = k_2 te^{-2t} \qquad \text{...(4)}$$

Diff. w.r.t. 't'

$$\frac{dv(t)}{dt} = k_2 te^{-2t}(-2) + k_2 e^{-2t}$$

put t = 0

$$\frac{dv(0)}{dt} = +k_2$$

To find $\frac{dv(0)}{dt}$, consider equation (1) put t = 0

$$0.5\frac{dv(0)}{dt} + \frac{0}{0.5} + 0 = 2$$

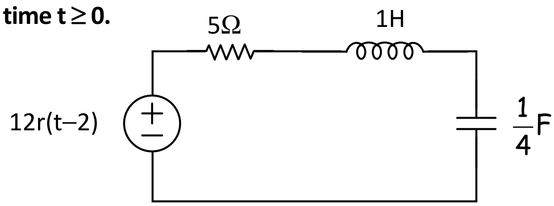
$$\therefore \quad \frac{dv(0)}{dt} = \frac{2}{0.5} = 4 \text{ v/s}$$

$\therefore \quad 4 = k_2 \quad \Rightarrow \quad k_2 = 4$

$\therefore$ equation (4) becomes

$v(t) = 4t\,e^{-2t} \qquad \therefore \quad t > 0$

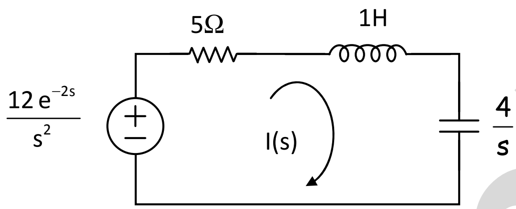
Q.4(a) Determine current in a circuit for time t ≥ 0. **[10]**



Ans.: As input is delayed and no initial conditions are given

$\therefore$ $i_L(0^-) = 0A$ & $V_C(0^-) = 0V$

Then the Laplace transformed network is given as,



By applying KVL to Loop

$$\frac{12\,e^{-2s}}{s^2} - 5I(s) - SI(s) - \frac{4}{s}I(s) = 0$$

$$\therefore \left(5 + s + \frac{4}{s}\right)I(s) = \frac{12e^{-2s}}{s^2}$$

$$\therefore I(s) = \frac{12\,e^{-2s}}{s^2(s^2+5s+4)}\cdot s$$

$$\therefore I(s) = \frac{12\,e^{-2s}}{s(s+1)(s+4)}$$

Consider,

$$\frac{12}{s(s+1)(s+4)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+4}$$

$$A\Big|_{s=0} = s\left[\frac{12}{s(s+1)(s+4)}\right] = \frac{12}{(1)(4)} = 3 \qquad \therefore A = 3$$

$$B\Big|s = -1 = (s+1)\left[\frac{12}{s(s+1)(s+4)}\right] = \frac{12}{(-1)(3)} = -4 \qquad \therefore B = -4$$

$$C\Big|_{s=-4} = (s+4)\left[\frac{12}{s(s+1)(s+4)}\right] = \frac{12}{-4(-3)} = 1 \qquad \therefore C = 1$$

$$\therefore I(s) = e^{-2s}\left[\frac{3}{s} - \frac{4}{(s+1)} + \frac{1}{s+4}\right]$$

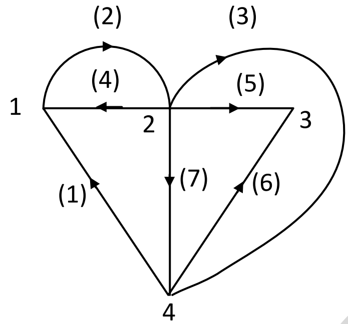
$$\therefore I(s) = \frac{3e^{-2s}}{s} - \frac{4e^{-2s}}{s+1} + \frac{e^{-2s}}{s+4}$$

By taking I.L.T.

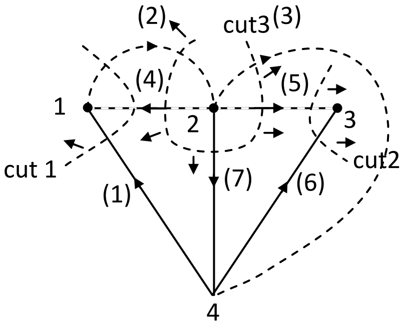
$$i(t) = 3u(t-2) - 4e^{-(t-2)}\cdot u(t-2) + e^{-4(t-2)}\,u(t-2)$$

$$\therefore i(t) = 3u(t-2) - 4e^{-(t-2)}\cdot u(t-2) + e^{-u(t-2)}\,u(t-2)$$

Q.4(b) For given graph find **[10]**
(i) Incidence matrix
(ii) tie set matrix
(iii) f-cut set matrix



Ans.:



(i) Incidence matrix [A]
To find complete incidence matrix can be written as

[Aa] =

Nodes \ Branch	1	2	3	4	5	6	7
1	−1	1	0	−1	0	0	0
2	0	−1	1	1	1	0	1
3	0	0	0	0	−1	−1	0
4	1	0	−1	0	0	1	−1

$$[A] = \begin{bmatrix} -1 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & -1 & 0 \end{bmatrix}$$

(ii) Tieset Matrix [B]
Consider branches 1, 7 & 6 as twing and rest all links forms loops.
Loop1{4, 1, 7}
Loop2{2, 1, 7}
Loop4{5, 6, 7}
Loop5{3, 7}

[B] =

Loop \ branch	1	2	3	4	5	6	7
1	−1	0	0	1	0	0	−1
2	1	1	0	0	0	0	1
3	0	0	0	0	1	−1	−1
4	0	0	1	0	0	0	−1

(iii) f-cutset matrix [Q]

cut1 : {1, 4, 2}

cut2 : {6, 5}

cut3 : {7, 2, 3, 4, 5}

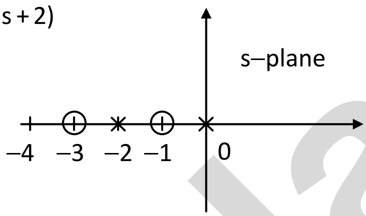
f − cut \ branch	1	2	3	4	5	6	7
1	1	−1	0	1	0	0	0
2	0	0	0	0	1	1	0
3	0	−1	1	1	1	0	1

Q.5(a) Realize the foster I and foster II forms of the following impedance function [10]

$$\mathbf{Z(s) = \frac{(s+1)(s+3)}{s(s+2)}}$$

Ans.: $Z(s) = \dfrac{(s+1)(s+3)}{s(s+2)}$

pole-zero plot.



As pole is closer to jω axis as pole at origin & zero at −1, therefore given function si realized by R-C components.

- Foster – I Form :

Let $z(s) = \dfrac{(s+1)(s+3)}{s(s+2)}$

$$z(s) = \frac{s^2+4s+3}{s^2+2s}$$

As Degree of numerator & denominator is same, therefore division is carried out.

$$\begin{array}{r|l} & 1 \\ s^2+2s & s^2+4s+3 \\ & \underline{s^2+2s} \\ & \quad 2s+3 \end{array}$$

$$\therefore \quad z(s) = 1+\frac{2s+3}{s^2+2s}$$

$$\therefore \quad z(s) = 1+\frac{2s+3}{s(s+2)}$$

$$\therefore \quad z(s) = 1+\frac{k_1}{s}+\frac{k^2}{(s+2)}$$

$$k_1 = s \cdot z(s)\big|_{s=0} = \frac{(1)(3)}{2} = \frac{3}{2}$$

$$k_2 = (s+2)\,z(s)\big|_{s=-2} = \frac{(-2+1)(-2+3)}{-2} = \frac{1}{2}$$

$$\therefore \quad z(s) = 1 + \frac{3/2}{s} + \frac{1/2}{s+2}$$

The first term represents resistor of 1Ω. The second term represent capacitor of $\frac{2}{3}$F

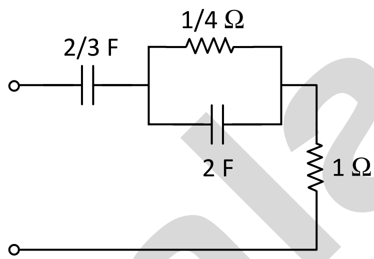
The third term represents the impedance of parallel RC circuit for which,

$$Z_{RC}(s) = \frac{1/Ci}{s + \frac{1}{RiCi}}$$

$\therefore$ By comparing C = 2F

$$R = \frac{1}{4}\Omega$$

Foster–I Realization is given as,



- **Forster – II Form :**

  The foster II form is obtained by the partial fraction expansion of admittance function $\frac{Y(s)}{s}$.

$$\text{Let } Y(s) = \frac{1}{z(s)} = \frac{s(s+2)}{(s+1)(s+3)}$$

$$\therefore \quad \frac{Y(s)}{s} = \frac{(s+2)}{(s+1)(s+3)}$$

$$\therefore \quad \frac{Y(s)}{s} = \frac{k_1}{s+1} + \frac{k_2}{s+3}$$

$$\therefore \quad k_1 = (s+1)\cdot\frac{Y(s)}{s}\bigg|_{s=-1} = \frac{1}{2}$$

$$k_2 = (s+3)\cdot\frac{Y(s)}{s}\bigg|_{s=-3} = \frac{1}{2}$$

$$\therefore \quad \frac{Y(s)}{s} = \frac{1/2}{s+1} + \frac{1/2}{s+3}$$

$\therefore \quad Y(s) = \dfrac{\frac{1}{2}s}{s+1} + \dfrac{\frac{1}{2}s}{s+3}$

There two terms represents the admittance of a series RC circuit. For series RC circuit.

$$Y_{RC}(s) = \frac{\left(\frac{1}{Ri}\right)s}{s + \frac{1}{RiCi}}$$

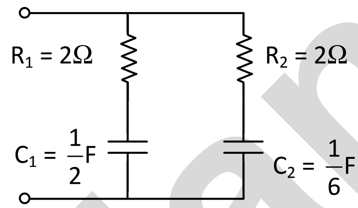
$\therefore$ By direct comparing

$R_1 = 2\Omega \qquad C_1 = \frac{1}{2}F$

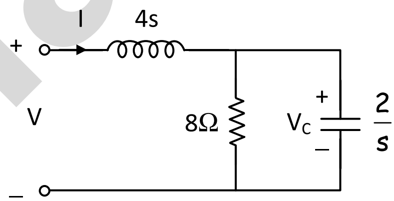
$R_2 = 2\Omega \qquad C_2 = \frac{1}{6}F$

Foster – II Realization

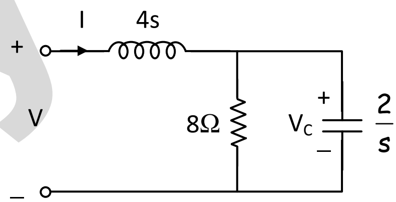
$\therefore$



Q.5(b) For the given circuit, determine $\dfrac{V_c}{V}$ & draw the pole-zero plot. **[5]**



Ans.: To Determine $\dfrac{V_c}{V}$



$8\Omega \;||\; \dfrac{2}{s}$

$$\therefore \quad \frac{8.\frac{2}{s}}{8+\frac{2}{s}} = \frac{16}{8s+2} = \frac{8}{1+4s}$$

$$\text{Let } V = \left[4s + \frac{8}{1+4s}\right]I$$

$$\therefore \quad V = \left[\frac{4s+16s^2+8}{1+4s}\right]I$$

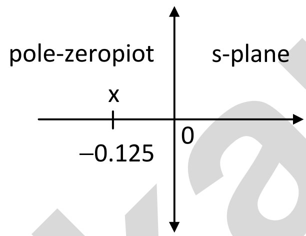
$$\therefore \quad V = \left[\frac{16s^2+4s+8}{1+4s}\right]I \qquad \text{...(1)}$$

Also $\quad V_C = \left[\dfrac{8}{1+4s}\right] I$ ...(2)

$$\therefore \quad \frac{V_C}{V} = \frac{\left[8 | 1+4s\right] I}{\left[\dfrac{16s^2+4s+8}{1+4s}\right] I}$$

$$\therefore \quad \frac{V_C}{V} = \frac{8}{16s^2+4s+8}$$

$$\therefore \quad \frac{V_C}{V} = \frac{2}{4s^2+s+2}$$



For polezero plot poles are at

$s_1 = 0.125 \quad \& \quad s_2 = -0.125$

No zeros

Q.5(c) Check if the following function is a positive real function. **[5]**

$$\mathbf{F(s) = \frac{2s^2 + 2s^2 + 3s + 2}{s^2 + 1}}$$

Ans.: To check for positive real function

$$F(s) = \frac{2s^3 + 2s^2 + 3s + 2}{s^2 + 1}$$

Step I :

To test N^r & D^r polynomial are Hurwitz or not

$$\therefore \quad F(s) = \frac{N(s)}{D(s)}$$

$\therefore \quad N(s) = 2s^3 + 2s^2 + 3s + 2 \qquad D(s) = s^2 + 1$

To check for Hurwitz polynomial

Consider, $N(s) = 2s^3 + 2s^2 + 3s + 2$

s^3	2	3
s^2	2	2
s^1	1	
s_0	2	

All the term's in first column of Routh array are positive, therefore the given polynomial N(s) is Hurwitz.

Consider $\quad D(s) = s^2 + 1$

$\quad D'(s) = 2s$

poles at $s^2 = -1$

$s = \pm j$

s^2	1	1
s	2	
s^0	1	

$\therefore \quad$ D(s) is also Hurwitz polynomial.

Step II :

As poles are on jw axis. Hence residue test is carried out.

Let N^r power is greater than D^r power therefore N^r is divided by D^r

$$s^2+1\overline{)2s^3+2s^2+3s+2}(2s+2$$
$$\underline{2s^3 \qquad +2s}$$
$$2s^2+s+2$$
$$\underline{2s^2 \quad +2}$$
$$s$$

$\therefore \quad F(s) = 2s+2+\dfrac{s}{s^2+1}$

Using partial fraction expansion.

$$F(s) = 2s+2+\frac{A_1}{s+j}+\frac{A_2}{s-j}$$

Consider, $\dfrac{s}{s^2+1} = \dfrac{A_1}{s+j}+\dfrac{A_2}{s-j}$

$$A_1\big|s=-j = (s+j)\cdot\frac{s}{(s+j)(s-j)}\bigg|_{s=-j} = \frac{-j}{-j-j} = \frac{1}{2}$$

$$A_1 = \frac{1}{2}$$

$$A_2 = (s-j)\frac{s}{(s+j)(s-j)}\bigg|_{s=j} = \frac{j}{j+j} = \frac{1}{2}$$

$\therefore \quad A_2 = \dfrac{1}{2}$

Thus the residues are real and positive.

Step III : Consider even part of N(s) = $m_1 = 2s^2 + 2$

odd part of N(s) = $n_1 = 2s^3 + 3s$

even part of D(s) = $m_2 = s^2 + 1$

odd part of b(s) = $n_2 = 0$

$\therefore \quad A(w^2) = m_1 . m_2 - n_1 \cdot n_2 \qquad = (2s^2 + 2)(s^2 + 1) - (2s^3 + 3)(0)$

$A(w^2) = 2s^4 + 4s^2 + 2$

put s = jw

$A(w^2) = 2w^4 - 2w^2 + 2$

$\therefore \quad A(w^2) \geq 0$ for all $w \geq 0$

$\therefore$ All three conditions are satisfied, therefore the function is positive real function.

Q.6(a) Check whether the following functions are prf or not : **[10]**

$$\mathbf{F(S)} = \frac{\mathbf{2S^4+7S^3+11S^2+12S+4}}{\mathbf{S^4+5S^3+9S^2+11S+6}}$$

Ans.: $F(S) = \dfrac{2S^4+7S^3+11S^2+12S+4}{S^4+5S^3+9S^2+11S+6} = \dfrac{N(S)}{D(S)}$

$N(S) = 2S^4 + 7S^3 + 11S^2 + 12S + 4$

S^4	2	11	4
S^3	7	12	–
S^2	$\frac{53}{7}$	4	–
S^1	$\frac{440}{53}$	–	–
S^0	4	–	–

N(S) is Hurwitz's polynomial.

$D(S) = S^4 + 5S^3 + 9S^2 + 11S + 6$

S^4	1	9	6
S^3	5	11	–
S^2	$\frac{34}{5}$	6	–
S^1	$\frac{112}{17}$	–	–
S^0	6	–	–

D(S) is Hurwitz's polynomial.

Determination of poles,

$D(S) = 0$

$\therefore \quad S^4 + 5S^3 + 9S^2 + 11S + 6 = 0$

Using synthetic division,

–1	1	5	9	11	6
		–1	–4	–5	–6
	1	4	5	6	0

$\therefore \quad S^4 + 5S^3 + 9S^2 + 11S + 6 = (S + 1)(S^3 + 4S^2 + 5S + 6)$

$= (S + 1)(S + 3)(S + 0.5 + j\,1.32)(S + 0.5 - j\,1.32)$

Poles are at $-1, -3, -0.5 - j\,1.32$ and $+0.5 - j\,1.32$

i.e. roots are complex, real and –ve.

Hence, residuals are not required.

Checking of A(w) function,

where $A(w) = m_1m_2 - n_1n_2$

$m_1 = (2S^4 + 11S^2 + 4)$ $\quad m_2 = (S^4 + 9S^2 + 6)$

$n_1\ (7S^3 + 12S)$ $\quad n_2 = (5S^3 + 11S)$

$A(w) = (2S^4 + 11S^2 + 4)(S^4 + 9S^2 + 6) - (7S^3 + 12S)(5S^3 + 11S)$

$= (2S^8 + 18S^6 + 12S^4 + 11S^6 + 99S^4 + 66S^2 + 4S^4 + 36S^2 + 24)$

$- (35S^6 + 77S^4 + 60S^4 + 132S^2)$

$= 2S^8 - 6S^6 - 22S^4 - 30S^2 + 24$

put S = jw

$\therefore \quad A(w) = 2(jw)^8 - 6(jw)^6 - 22(jw)^4 - 30(jw)^2 + 24$

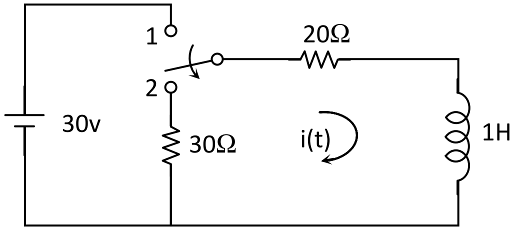
$= 2w^8 + 6w^6 - 22w^4 - 30w^2 + 24$

it is clear that for all values of w, $A(w) \geq 0$

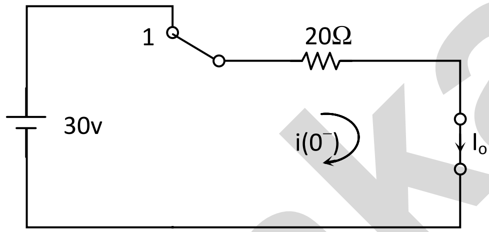
Hence, all conditions are satisfied.

$\therefore$ it is prf.

Q.6(b) **The network shown in Figure reaches a steady state with switch at position 1. At t = 0, the switch is changed from the position 1 to the position 2, Find the value of i, $\frac{di}{dt}$, $\frac{d^2i}{dt^2}$ at t = 0^+.** **[10]**



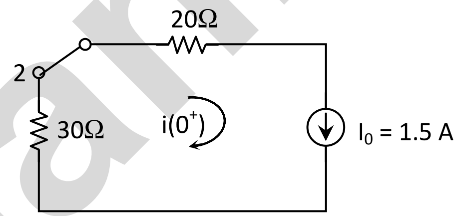
Ans.: Draw circuit for t < 0 (t = 0^-)



$I_0 = i(0^-) = \frac{30}{20}$

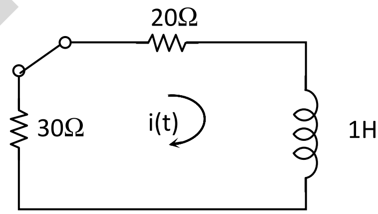
$= 1.5$ Amp

Draw circuit at time t = 0^+



$I_0 = i(0^+) = 1.5$ Amp

Draw circuit at time, t ≥ 0



Apply KVL to loop,

$$-30\, i(t) - 20\, i(t) - 1\frac{di(t)}{dt} = 0$$

$$\frac{di(t)}{dt} = -50\, i(t) \qquad \ldots (1)$$

put t = 0^+

$$\frac{di(0^+)}{dt} = -50\, i(0^+) = 50 \times 1.5 = 75 \text{ Amp/sec}$$

Differentiate (1) w.r.t. t

$$\frac{d^2 i(t)}{dt^2} = -50\frac{di(t)}{dt}$$

put time, t = 0^+

$$\frac{d^2 i(0^+)}{dt^2} = -50\frac{di(0^+)}{dt^2} = -50 \times 75 = -3750 \text{ Amp/sec}^2$$

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