**Prelim Paper** **Applied Mathematics - III**

**Time: 3 Hrs.]** **[Marks : 80**

**N.B.:** (1) Question No. 1 is compulsory.
(2) Attempt any three questions from the remaining.
(3) Figures to the right indicate full marks.

**1.** (a) Find Laplace transform of $te^{3t}\cos t$. **[5]**

(b) Prove that $\overline{f} = (x + 2y + az)i + (bx - 3y - z)j + (4x + cy + 2z)k$ is solenoidal and determine the constants a, b, c if $\overline{f}$ is irrotational. **[5]**

(c) Show that f(z) = sinh z is analytic. Hence find its derivative. **[5]**

(d) Prove that $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\cdot\cos x$ **[5]**

**2.** (a) Show that the function $w = \frac{4}{z}$ transforms the straight line x = c in the z-plane into circle is w-plane. Find its centre and radius. **[6]**

(b) Show that $\int_0^\infty e^{-t}\int_0^t \frac{\sin u}{u}\,du\,dt = \frac{\pi}{4}$ **[6]**

(c) Obtain fourier series for $f(x) = \begin{cases} 1+\frac{2x}{\pi} & -\pi \le x \le 0 \\ 1-\frac{2x}{\pi} & 0 \le x \le \pi \end{cases}$ **[8]**

Hence deduce that $\frac{\pi^2}{8} = \frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots$

**3.** (a) Evaluate by Green's Theorem $\int_C \left(e^{-x}\sin y\,dx + e^{-x}\cos y\,dy\right)$ where C is rectangle with vertices (0, 0), (π, 0), $(\pi, \frac{\pi}{2})$, $(0, \frac{\pi}{2})$ **[6]**

(b) Prove that $J_2'(x) = \left(1-\frac{4}{x^2}\right)J_1(x)+\frac{2}{x}J_0(x)$ **[6]**

(c) Solve the differential equation **[8]**

$\frac{dy}{dx}+2y+\int_0^t y\,dt = \sin t$ using Laplace transform give y(0) = 1

**4.** (a) Find orthogonal trajectory to the family of curves $e^{-x}\cos y + xy$ = constant in X – Y plane. **[6]**

(b) Show that $\cos x = \frac{8}{\pi}\sum_{m=1}^{\infty}\frac{m}{4m^2-1}\sin(2mx)$ If $0 < x < \pi$ **[6]**

(c) Find Bilinear transformation which maps the points 1, i, –1 onto the points i, 0, –i. Hence find fixed points and image of $|z| < 1$. **[8]**

**5.** (a) Use Stoke's Theorem to evaluate $\int_c \overline{f} \cdot \overline{dr}$ **[6]**

Where $\overline{f} = yi + zj + xk$ and c is boundary of the surface $x^2 + y^2 = 1 - z$, $z > 0$.

(b) If $f(x) = c_1 \phi_1(x) + c_2\phi_2(x) + c_3 \phi_3(x)$, where $c_1$, $c_2$, $c_3$ are constant and $\phi_1$, $\phi_2$, $\phi_3$ are orthonormal functions, on the set (a, b). **[6]**

Show that $\int_a^b [f(x)]^2\, dx = c_1^2 + c_2^2 + c_3^2$

(c) Find inverse Laplace Transform of **[8]**

(i) $\log\left(1 + \frac{\alpha^2}{s^2}\right)$ (ii) $\frac{e^{-s}}{s^2 + s + 1}$

**6.** (a) Obtain complex form of fourier series for $F(x) = e^{\alpha x}$, in $(-\pi, \pi)$ where a is not an integer. **[6]**

(b) Prove that $\overline{f} = (ye^{xy} \cos z)i + (xe^{xy} \cos z)j + (-e^{xy} \sin z)k$ is irotational . **[6]**
Also find scalar potential $\phi$ and work done in moving particle from (0, 0, 0) to $(-1, 2, \pi)$

(c) Find imaginary part of analytic function whose real part is $e^{2x}(x \cos 2y - y \sin 2y)$, Also verify that v is harmonic function. **[8]**

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