

Prelim Paper

Time: 3 Hrs.]

Applied Mathematics - III

[Marks : 80

- N.B.:** (1) Question No. 1 is compulsory.
 (2) Attempt any three questions from the remaining.
 (3) Figures to the right indicate full marks.

1. (a) Find Laplace transform of $te^{3t} \cos t$. [5]
 (b) Prove that $\vec{f} = (x + 2y + az)\mathbf{i} + (bx - 3y - z)\mathbf{j} + (4x + cy + 2z)\mathbf{k}$ is solenoidal and determine the constants a, b, c if \vec{f} is irrotational. [5]
 (c) Show that $f(z) = \sinh z$ is analytic. Hence find its derivative. [5]
 (d) Prove that $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cdot \cos x$ [5]
2. (a) Show that the function $w = \frac{4}{z}$ transforms the straight line $x = c$ in the z-plane into circle in w-plane. Find its centre and radius. [6]
 (b) Show that $\int_0^{\infty} e^{-t} \int_0^t \frac{\sin u}{u} du dt = \frac{\pi}{4}$ [6]
 (c) Obtain fourier series for $f(x) = \begin{cases} 1 + \frac{2x}{\pi} & -\pi \leq x \leq 0 \\ 1 - \frac{2x}{\pi} & 0 \leq x \leq \pi \end{cases}$ [8]
 Hence deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$
3. (a) Evaluate by Green's Theorem $\int_C (e^{-x} \sin y dx + e^{-x} \cos y dy)$ where C is rectangle [6]
 with vertices $(0, 0), (\pi, 0), (\pi, \frac{\pi}{2}), (0, \frac{\pi}{2})$
 (b) Prove that $J_2'(x) = \left(1 - \frac{4}{x^2}\right)J_1(x) + \frac{2}{x}J_0(x)$ [6]
 (c) Solve the differential equation $\frac{dy}{dx} + 2y + \int_0^t y dt = \sin t$ using Laplace transform [8]
 give $y(0) = 1$
4. (a) Find orthogonal trajectory to the family of curves $e^{-x} \cos y + xy = \text{constant}$ in X - Y plane. [6]
 (b) Show that $\cos x = \frac{8}{\pi} \sum_{m=1}^{\infty} \frac{m}{4m^2 - 1} \sin(2mx)$ [6]
 If $0 < x < \pi$
 (c) Find Bilinear transformation which maps the points 1, i, -1 onto the points i, 0, -i. Hence find fixed points and image of $|z| < 1$. [8]

5. (a) Use Stoke's Theorem to evaluate $\int_c \bar{f} \cdot d\bar{r}$ [6]

Where $\bar{f} = y\mathbf{i} + z\mathbf{j} + x\mathbf{k}$ and c is boundary of the surface $x^2 + y^2 = 1 - z, z > 0$.

- (b) If $f(x) = c_1 \phi_1(x) + c_2 \phi_2(x) + c_3 \phi_3(x)$, where c_1, c_2, c_3 are constant and ϕ_1, ϕ_2, ϕ_3 are orthonormal functions, on the set (a, b) . [6]

Show that $\int_a^b [f(x)]^2 dx = c_1^2 + c_2^2 + c_3^2$

- (c) Find inverse Laplace Transform of [8]

(i) $\log\left(1 + \frac{\alpha^2}{s^2}\right)$ (ii) $\frac{e^{-s}}{s^2 + s + 1}$

6. (a) Obtain complex form of fourier series for $F(x) = e^{ax}$, in $(-\pi, \pi)$ where a is not an integer. [6]

- (b) Prove that $\bar{f} = (ye^{xy} \cos z)\mathbf{i} + (xe^{xy} \cos z)\mathbf{j} + (-e^{xy} \sin z)\mathbf{k}$ is irrotational. [6]

Also find scalar potential ϕ and work done in moving particle from $(0, 0, 0)$ to $(-1, 2, \pi)$

- (c) Find imaginary part of analytic function whose real part is $e^{2x}(x \cos 2y - y \sin 2y)$, Also verify that v is harmonic function. [8]

