N.B.:   (1)   Question No. **1** is **compulsory**.
          (2)   Attempt any **THREE** of the remaining.
          (3)   Figures to the right indicate full marks.

1. (a) If any 11 numbers between 1 and 20 are chosen show that at least two of them will be multiplies of each other. **[5]**

   (b) A function, $f : R - \left\{\dfrac{7}{3}\right\} \to R - \left\{\dfrac{4}{3}\right\}$ is defined by $f(x) = \dfrac{4x-5}{3x-7}$, prove that f is bijective and find the rule for f^{-1}. **[5]**

   (c) Find $L\left[\dfrac{d}{dt}\left(\dfrac{1-\cos 2t}{t}\right)\right]$ **[5]**

   (d) Prove that there does not exist an analytic function whose imaginary part is
   $3x^2 + \sin x + y^2 + 5y + 4$ **[5]**

2. (a) Evaluate : $\displaystyle\int_{t=0}^{\infty} e^{-3t}\left(\dfrac{\cos(7t)-\cos(11t)}{t}\right)dt$ **[6]**

   (b) Find $L^{-1}\left[\dfrac{s^2+2s+3}{(s^2+2s+10)\,(s^2+2s+17)}\right]$ **[6]**

   (c) Find the bilinear Transformation which maps the points 2, i, −2 on to the points 1, i, −1. Also find image of |z| = 1 of z-plane to w-plane. **[8]**

3. (a) Prove that : $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$ **[6]**
   (b) Draw the Hasse diagram of D_{105}. **[6]**
   (c) Find Laplace Transformation of the following : **[8]**
   (i) $t\,e^{3t}\,\text{erf}\,(5\sqrt{t})$
   (ii) $\sin t\,H(t) + (\cos t - \sin t)\,H(t - \pi)$

4. (a) A family consisting of an old man, 6 adults and 4 children is to be seated in a row for dinner. The children wish to occupy two seats at each end and the old man refuse to have a child on either side of him. In how many ways can the seating arrangement be made for the dinner? **[6]**
   (b) Find the analytic function f(z) = u + iv in terms of z if $u - v = (x - y)\,(x^2 + 4xy + y^2)$. **[6]**
   (c) Solve $\dfrac{d^3y}{dt^3} - 2\dfrac{d^2y}{dt^2} + 5\dfrac{dy}{dt} = 0$ with y(0) = 0, y′(0) = 0, y″(0) = 1. **[8]**

5. (a) Find $L^{-1}\left[\dfrac{s}{(s^2+3^2)(s^2+5^2)}\right]$ using convolution Theorem. **[6]**
   (b) What is the chance of throwing ten with four dice? **[6]**

(c) In a certain examination there are multiple choice questions. There are four possible answers to each questions and one of them is correct. An intelligent student can solve 90% questions correctly by reasoning and for the remaining 10% questions he give answer by guessing. A week student can solve 20% question correctly by reasoning and for the remaining 80% questions he gives answer by guessing. An intelligent student gets the correct answer. What is the probability that he was guessing. **[8]**

6. (a) A can hit a target 2 times in 5 shots, B 3 times in 4 shots, C 2 times in 3 shots. They fire a volley. What is the probability that at least 2 shots hit the target? **[6]**

(b) Find $L^{-1}\left(\tan^{-1}\left(\dfrac{2}{s^2} \right) \right)$ **[6]**

(c) If R is the relation on the set of integers such that aRb if and only if 2a + 3b is divisible by 5. Find the equivalence classes. **[8]**

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