Applied Mathematics - III

Time : 3 Hrs.] Prelim Paper Solution **[Marks : 80**

Q.1(a) If any 11 numbers between 1 and 20 are chosen show that at least two of them will be multiples of each other. **[5]**

Ans.: Any natural number can be expressed $b = 2^k . m$
where 'm' is known as odd part of 'n'. and $k \geq 0$.
By this way we find there are 10 odd parts in the set {1, 2, … 20}
$\because$ there are 10 odd parts (10 pigeon holes) and chosen numbers are 11 (11 pigeon).
$\Rightarrow$ Pigeonhole principle is applicable

By pigeon hole principle one pigeonhole must have atleast two pigeons
$\therefore$ Two natural numbers must have same odd part
Let n_1 & n_2 have same odd part 'n'

$$n_1 = 2^{k_1} m \qquad n_2 = 2^{k_2} m$$

There are two possibilities
(i) $k_1 < k_2 \quad \Rightarrow \quad n_1 \mid n_2$
(ii) $k_1 > k_2 \quad \Rightarrow \quad n_2 \mid n_1$ either way one is multiple of other.

Q.1(b) A function, $f : R - \left\{\frac{7}{3}\right\} \to R - \left\{\frac{4}{3}\right\}$ is defined by $f(x) = \frac{4x-5}{3x-7}$, prove that f is bijective and find the rule for f^{-1}. **[5]**

Ans.: $\because$ $f : R - \left\{\frac{7}{3}\right\} \to R - \left\{\frac{4}{3}\right\}$

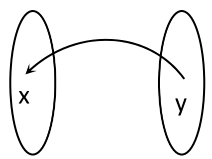
is defined as $f(x) = \frac{4x-5}{3x-7}$

(i) Injective
Consider $f(x_1) = f(x_2)$

$\therefore \frac{4x_1-5}{3x_1-7} = \frac{4x_2-5}{3x_2-7}$

or $(4x_1-5)(3x_2-7) = (3x_1-7)(4x_2-5)$
or $12x_1x_2 - 28x_1 - 15x_2 + 35 = 12x_1x_2 - 28x_1 - 15x_2 + 35$
or $-13x_1 = -13x_2$
or $x_1 = x_2$
$\therefore$ f is injective

(ii) Surjective



(domain) (codomain)

$R - \{7/3\}$ $R - \{4/3\}$

Consider an arbitrary element y in R – {4/3} (codomain)

Let $y = f(x)$

$$y = \frac{4x-5}{3x-7}$$

or $3xy - 7y = 4x - 5$

or $3xy - 4x = 7y - 5$

or $$x = \frac{7y-5}{3y-4}$$

$\forall\ y \in R - \{4/3\}$ (codomain) $\exists$ pre image $x \in R - \{7/3\}$ (domain)

$\Rightarrow$ Range of f = codomain

$\Rightarrow$ f is surjective

$\because$ f is injective & surjective both

$\therefore$ f is bijective

$\therefore$ f^{-1} exists.

Let $y = f(x) \Rightarrow x = f^{-1}(y)$

$$y = \frac{4x-5}{3x-7}$$

$\therefore\ 3xy - 7y = 4x - 5$

$$\Rightarrow x = \frac{7y-5}{3y-4} = f^{-1}(y)$$

$\therefore$ The rule for f^{-1} is

$$f^{-1}(x) = \frac{7x-5}{3x-4}$$

Q.1(c) Find $L\left[\frac{d}{dt}\left(\frac{1-\cos 2t}{t}\right)\right]$ **[5]**

Ans.: $L\left[\frac{d}{dt}\left(\frac{1-\cos 2t}{t}\right)\right]$

Let $f(t) = \frac{1-\cos 2t}{t}$

$$L[f(t)] = f(\overline{s}) = L\left[\frac{1-\cos 2t}{t}\right] = \int_s^\infty L[1-\cos 2t]\,ds = \int_s^\infty \left[\frac{1}{s} - \frac{s}{s^2+4}\right] ds$$

$$= \left[\log s - \frac{1}{2}\log(s^2+4)\right]_s^\infty = \frac{1}{2}\left[\log s^2 - \log(s^2+4)\right]_s^\infty = \frac{1}{2}\left[\log\frac{s^2}{s^2+4}\right]_s^\infty$$

$$= \frac{1}{2}\left\{\log\frac{1}{1+(4/s^2)}\right\}_s^\infty = \frac{1}{2}\left\{\log(1) - \log\frac{s^2}{s^2+\phi}\right\} = -\frac{1}{2}\log\frac{s^2}{s^2+4}$$

$$= \log\sqrt{\frac{s^2+4}{s^2}}$$

$$f(\overline{s}) = \log\frac{\sqrt{s^2+4}}{\sqrt{s^2}} \qquad = \log\frac{\sqrt{s^2+4}}{s}$$

$$f(0) = \lim_{t\to 0} f(t) = \lim_{t\to 0}\frac{1-\cos 2t}{t} \Rightarrow \left(\frac{0}{0}\right)$$

$$= \lim_{t\to 0}\frac{0+2\sin 2t}{1} = 0$$

But $L[f'(t)] = s\bar{f}(s) - f(0)$

$$L\left[\frac{d}{dt}\frac{(1-\cos 2t)}{t}\right] = s\log\left(\frac{\sqrt{s^2+4}}{s}\right) - 0$$

Q.1(d) Prove that there does not exist an analytic function whose imaginary part is $3x^2 + \sin x + y^2 + 5y + 4$ [5]

Ans.: Let $V = 3x^2 + \sin x + y^2 + 5y + 4$

$V_x = 6x + \cos x$

$V_{xx} = 6 - \sin x$

$V_y = 2y + 5$

$V_{yy} = 2$

$\therefore$ $V_{xx} + V_{yy} = 8 - \sin x \neq 0$

$\therefore$ V is not Harmonic function

$\therefore$ $V = 3x^2 + \sin x + y^2 + 5y + 4$ is not a imaginary part of analytic function f(z) = 4 + iv

Q.2(a) Evaluate : $\int_{t=0}^{\infty} e^{-3t}\left(\frac{\cos(7t)-\cos(11t)}{t}\right)dt$ [6]

Ans.:

$$J = \int_{t=0}^{\infty} e^{-3t}\left[\frac{\cos(7t)-\cos(11t)}{t}\right]dt$$

$$= \int_{t=0}^{\infty} e^{-st}\left[\frac{\cos(7t)-\cos(11t)}{t}\right]dt \quad \text{where } s = 3$$

$$= L\left[\frac{\cos(7t)-\cos(11t)}{t}\right] \quad \text{where } s = 3 \qquad \ldots(1)$$

Now $L[\cos(7t)-\cos(11t)] = \frac{s}{s^2+7^2} = \frac{s}{s^2+11^2}$

$$\therefore L\left[\frac{\cos(7t)-\cos(11t)}{t}\right] = \int_{s=3}^{\infty}\left(\frac{s}{s^2+7^2}-\frac{s}{s^2+11^2}\right)ds$$

$$= \frac{1}{2}\log(s^2+7^2) - \log(s^2+11^2)\Big|_{s=3}^{\infty}$$

$$= \frac{1}{2}\log\left(\frac{s^2+7^2}{s^2+11^2}\right)\Bigg|_{s=3}^{\infty} = \frac{1}{2}\log\left(\frac{s^2+11^2}{s^2+7^2}\right)$$

$$\therefore L\left[\frac{\cos(7t)-\cos(11t)}{t}\right]_{s=3} = \frac{1}{2}\log\left(\frac{9+121}{9+49}\right) = \frac{1}{2}\log\left(\frac{130}{58}\right)$$

$$= \frac{1}{2}\log\left(\frac{65}{29}\right) \qquad \ldots(2)$$

From (1) & (2) $I = \frac{1}{2}\log\left(\frac{65}{29}\right)$

Q.2(b) **Find** $L^{-1}\left[\dfrac{s^2+2s+3}{(s^2+2s+10)\,(s^2+2s+17)}\right]$ **[6]**

Ans.: Let $s^2 + 2s = x$

$$\therefore \quad \frac{x+3}{(x+10)(x+17)} = \frac{\frac{-10+3}{-10+17}}{x+10}+\frac{\frac{-17+3}{-17+10}}{21+17} = \frac{\frac{-7}{7}}{x+10}+\frac{\frac{-14}{-7}}{x+17} = \frac{2}{x+17}-\frac{1}{x+10}$$

$$\therefore \quad \frac{s^2+2s+3}{(s^2+2s+10)(s^2+2s+17)} = \frac{2}{s^2+2s+17}-\frac{1}{s^2+2s+10}$$

$$= \frac{2}{s^2+2s+1+16}-\frac{1}{s^2+2s+1+9}$$

$$= \frac{2}{4}\left[\frac{4}{(s+1)^2+4^2}\right]-\frac{1}{3}\frac{3}{(s+1)^2+3^2}$$

$$\therefore \quad L^{-1}\left[\frac{s^2+2s+3}{(s^2+2s+10)(s^2+2s+17)}\right] = \frac{1}{2}L^{-1}\frac{4}{(s+1)^2+4^2}-\frac{1}{3}L^{-1}\left[\frac{3}{(s+1)^2+3^2}\right]$$

$$= \frac{1}{2}e^{-1t}L^{-1}\left(\frac{4}{s^2+4^2}\right)-\frac{1}{3}e^{-1t}\,L^{-1}\left(\frac{3}{s^2+3^2}\right)$$

$$= \frac{1}{2}e^{-1t}\sin 4t-\frac{1}{3}e^{-1t}\sin(3t)$$

$$\therefore \quad L^{-1}\left[\frac{(s^2+2s+3)}{(s^2+2s+100)(s^2+10s+17)}\right] = e^{-1t}\left[\frac{\sin(4t)}{2}-\frac{\sin(3t)}{3}\right]$$

Q.2(c) **Find the bilinear Transformation which maps the points 2, i, –2 on to the points 1, i, –1. Also find image of |z| = 1 of z-plane to w-plane.** **[8]**

Ans.: Z–plane $z_1 = 2,\ z_2 = 2,\ z_3 = 1^\circ,\ z_4 = -2$

$w_1 = w,\ w_2 = 1,\ w_3 = i, w_4 = -1$

By cross ratio, we have,

$$\frac{(z_1-z_2)(z_3-z_4)}{(z_2-z_3)(z_4-z_1)} = \frac{(w_1-w_2)(w_3-w_4)}{(w_2-w_3)(w_4-w_1)}$$

$$\frac{(z-2)(i+2)}{(2-i)(-2-z)} = \frac{(w-1)(i+1)}{(1-i)(-1-w)}$$

$$\frac{iz+2z-2i-4}{-4-2z+2i+iz} = \frac{iw+w-i-1}{-1-w+i+iw}$$

$-iz - iw - z - wz - 2z - 2wz + 2iz + 2iwz + 2i + 2iw + 2 + 2w + 4 + 4w - 4i - 4iw$ $= -4iw - 4w + 4i + 4 - 2iwz - 2wz + 2iz + 2z - 2w + 2iw + 2 - 2i - wz + iwz + z - iz$

$-iwz - z - 2z + 2iwz + 2i + 2w + 4w - 4i$ $= -4w + 4i - 2iwz + 2z - 2w - 2i + iwz + z$

$iwz - 3z + 6w - 2i = -6w + 2i - iwz + 3z$

$iwz + 6w + 6\omega + iwz = 2i + 3z + 3z + 2i$

$2iwz + 12w = 4i + 6z$

$iwz + 6w = 2i + 3z$

$$w(iz + 6) = 2i + 3z$$

$$w = \frac{2i+3z}{iz+6}$$

Q.3(a) Prove that : $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$ **[6]**

Ans.:

$$(A - B) \cup (B - A) = (A \cap \overline{B}) \cup (B \cap \overline{A})$$

$$= [(A \cap \overline{B}) \cup B] \cap [(A \cap \overline{B}) \cup \overline{A}] \quad [\because \text{ Distributive law}]$$

$$= [B \cup (A \cap \overline{B})] \cap [\overline{A} \cup (A \cap \overline{B})] \quad [\because \cup \text{ is commutative}]$$

$$= [(B \cup A) \cap (B \cup \overline{B})] \cap [(\overline{A} \cup A) \cap (\overline{A} \cup \overline{B})]$$

$[\because$ Distributive law$]$

$$= [(B \cup A) \cap U] \cap [U \cap (\overline{A \cap B})] \quad \text{[De'Morgan's ;aw]}$$

$$= (B \cup A) \cap \overline{A \cap B}$$

$$= (A \cup B) \cup \overline{A \cap B} \quad \because \text{ U is commutative}$$

$$= (A \cup B) - (A \cap B)$$

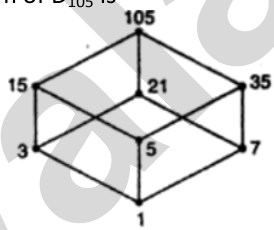
Q.3(b) Draw the Hasse diagram of D_{105}. **[6]**

Ans.: $105 = 5 \times 21 = 5 \times 3 \times 7 = 1 \times 3 \times 5 \times 7$

Let A be event of division of 105

$\therefore$ A = {1, 3, 5, 7, 15, 21, 35, 105}

$\therefore$ Required Hasse diagram of D_{105} is



Q.3(c) Find Laplace Transformation of the following : **[8]**

(i) $t\,e^{3t}\,\text{erf}\,(5\sqrt{t})$ **(ii) $\sin t\, H(t) + (\cos t - \sin t)\, H(t - \pi)$**

Ans.: (i) $t\,e^{3t}\,\text{erf}\,(5\sqrt{t})$

we know Let $f_{(out)} = \dfrac{a}{\sqrt{s+a^2}}$

$$\therefore \quad L\,\text{erf}\,(5\sqrt{t}) = \frac{5}{\sqrt{s+25}} = 5(s+25)^{-\frac{1}{2}}$$

$$\therefore \quad L\left(t\,\text{erf}\left(5\sqrt{t}\right)\right) = (-1)\frac{d}{ds}\left[5(s+25)^{\frac{-1}{2}}\right] = (-5)\left(\frac{-1}{2}\right)(s+25)^{-\frac{3}{2}}$$

$$L[t\,\text{erf}\,(5\sqrt{t})\,] = \frac{5}{2(s+25)^{\frac{3}{2}}}$$

$$L\left[e^{3t}\left[t\,\text{erf}\left(5\sqrt{t}\right)\right]\right] = \frac{5}{2[(s-3)+25]^{\frac{3}{2}}} = \frac{5}{2(s+22)^{\frac{3}{2}}}$$

(ii) sin t H(t) + (cos t – sin t) H(t – π)

$$\text{Lsin(t)} = \frac{1}{s^2+1^2}$$

$$L[H(t)\sin t] = L[H(t-0)\sin(t)] = e^{-os}\,L\sin(t+0) = e^{-os}\,L(\sin t)$$

$$\therefore\quad L[H(t)\sin t] = \frac{1}{s^2+1^2} \qquad \ldots(1)$$

$$L[H(t-\pi)(\cos t-\sin t)] = e^{-\pi s}L\left[\cos(t+\pi)-\sin(t+\pi)\right]$$

$$= e^{-\pi s}L\left[-\cos t+\sin t\right]$$

$$= e^{-\pi s}\,L(\sin t-\cos t)$$

$$= e^{-\pi s}\left(\frac{1}{s^2+1^2}-\frac{s}{s^2+1^2}\right)$$

$$= e^{-\pi s}\left(\frac{1-s}{s^2+1^2}\right) \qquad \ldots(ii)$$

$$\therefore\quad L\left[H(t)\sin t+H(t-\pi)(\cos t-\sin t)\right]$$

$$= L\left[H(t)\sin(t)\right]+L\left[H(t-\pi)(\cos t-\sin t)\right]$$

$$= \frac{1}{s^2+1^2}+e^{-\pi s}\left[\left(\frac{1-s}{s^2+1^2}\right)\right]$$

Q.4(a) A family consisting of an old man, 6 adults and 4 children is to be seated in a row for dinner. The children wish to occupy two seats at each end and the old man refuse to have a child on either side of him. In how many ways can the seating arrangement be made for the dinner? **[6]**

Ans.: The seating arrangement of 6 adults & 4 children is as follows

C C A O A O A O A O A O A C C

By given an old man refuse to have child on either side of him.

∴ For old man five places are available

∴ Required number of ways = $^4D_4 \times {}^6D_6 \times 5$ = 86400

Q.4(b) Find the analytic function f(z) = u + iv in terms of z if **[6]**

u – v = (x – y) (x^2 + 4xy + y^2)

Ans.: $u - v = x^3 + 4x^2y + xy^2 - x^2y - 4xy^2 - y^3$

u + iv = f(z) …(1)

iu – v = if(z) …(2)

Adding (1) & (2), we get

(u – v) + i(u + v) = (1+ i) f(z) = f(z)

Let f(z) = U + iv where U = u – v& V = u + v

Now $U = x^3 + 4x^2y + xy^2 - x^2y - 4xy^2 - y^3$

$$\frac{\partial U}{\partial x} = 3x^2 + 8xy + y^2 - 2xy - 4y^2 = 3x^2 + 6xy - 3y^2$$

$$\frac{\partial U}{\partial y} = 4x^2 + 2xy - x^2 - 8xy - 3y^2 = 3x^2 - 6xy - 3y^2$$

We know that

$$dv = \frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial y}dy$$

$$= -\frac{\partial U}{\partial y}dx + \frac{\partial U}{\partial x}dy \qquad (\because \text{ f(z) is analytic})$$

$$= \left(-4x^2 - 2xy + x^2 + 8xy + 3y^2\right)dx + \left(3x^2 + 8xy + y^2 - 2xy - 4y^2\right)dy$$

Integrating, We get

$$V = -x^3 + 3x^2y + 3xy^2 - y^3 + C$$

$\therefore$ F(z)= U + iV

$$= \left(x^3 + 4x^2y + xy^2 - x^2y - 4xy^2 - y^3\right) + i\left(-x^3 + 3x^2y + 3xy^2 - y^3 + C\right)$$

$$= \left(x^3 + 3x^2y - 3xy^2 - y^3\right) + i\left(-x^3 + 3x^2y + 3xy^2 - y^3\right) + C$$

$$= (x + iy)^3 + \left[i[x + iy]\right]^3 + C$$

$$= (x + iy)^3 - i(x + iy)^3 + C$$

$$= (1 - i)(x + iy)^3 + C$$

$$= (1 - i)z^3 + C$$

$\therefore$ (1–i) f(z) = (1– i) z^3 + C

$\therefore$ f(z) = $z^3 + C_1$ where $C_1 = \dfrac{C}{1-i}$

Q.4(c) Solve $\dfrac{d^3y}{dt^3} - 2\dfrac{d^2y}{dt^2} + 5\dfrac{dy}{dt} = 0$ with y(0) = 0, y′(0) = 0, y′′(0) = 1. **[8]**

Ans.: Given differential equation can be written as

$$y'''(t) - 2y''(t) + 5y'(t) = 0$$

$$\therefore \quad L\left(y'''(t)\right) - 2\,L[y''(t)] + 5\,L[y'(t)] = 0$$

$$s^3\overline{y}(s) - s^2y(0) - sy^1(0) - y''(0) - 2[s^2\overline{y}(s) - sy(0) - y'(0)] + 5[s\overline{y}(s) - y(0)] = 0$$

$\therefore$ y(0) = 0, y′(0) = 0, y′′(0) = 1

$$s^3\,\overline{y}\,(s) - 1 - 2s^2 + 5s\,\overline{y}\,(s) = 0$$

$$\overline{y}(s)\,s(s^2 - 2s + 5) = 1$$

$$\overline{y}(s) = \frac{1}{s\left(s^2 - y(s)\right)} = \frac{1}{s}\overline{f}(s) \qquad \ldots(1) \qquad \text{where } \overline{f}(s) = \frac{1}{s^2 - 2s + 5}$$

$$\therefore \quad L^{-1}\left(\overline{y}(s)\right) = L^{-1}\left[\frac{1}{s\left(s^2 - 2s + 5\right)}\right] = L^{-1}\left(\frac{1}{s}\overline{f}(s)\right) = \int_{u=0}^{t} f(u)du \qquad \ldots(2)$$

$$\text{where} \quad \overline{f}(s) = \frac{1}{s^2 - 2s + 5} = \frac{1}{s^2 - 2s + 1 + 4} = \frac{1}{2}\left[\frac{2}{(s-1)^2 + 2^2}\right]$$

$$\therefore \quad f(t) = L^{-1}\left(\overline{f}(s)\right) = \frac{1}{2}L^{-1}\left[\frac{2}{(s-1)^2 + 2^2}\right] = \frac{1}{2}e^{1-t}L^{-1}\left(\frac{2}{s^2 + 2^2}\right)$$

$$\therefore \quad f(s) = \frac{1}{2}e^{1t}\sin(2t) \qquad \ldots(3)$$

From (2) & (3)

$$L^{-1}\left(\overline{y}(s)\right) = \int_{u=0}^{t} \frac{e^{14}}{2} \sin(24)du = \frac{1}{2}\int_{u=0}^{t} e^{1u}\sin(2u)du$$

$$\therefore \quad y(t) = \frac{1^{1u}}{1^2+2^2}\left[1\sin(2u) - 2\cos(2u)\right]_{u=0}^{t}$$

$$y(t) = \frac{1}{10}\left[e^{t}\left[\sin 2t - 2\cos 2t\right] - 1\left[0-2\right]\right]$$

$$y(t) = \frac{1}{10}\left[e^{t}\left[\sin(nt) - 2\cos 2t\right] + 2\right]$$

Q.5(a) **Find** $L^{-1}\left[\frac{s}{(s^2+3^2)(s^2+5^2)}\right]$ **using convolution Theorem.** **[6]**

Ans.:

$$I = L^{-1}\left[\frac{s}{(s^2+3^2)(s^2+5^2)}\right] = L^{-1}\left[\left(\frac{s}{s^2+3^2}\right)\left(\frac{1}{s^2+s^2}\right)\right] = L^{-1}\left[\overline{f}_1(s)\cdot\overline{f}_2(s)\right]$$

$$I = \int_{u=0}^{t} f_1(u)\cdot f_2(2)du \qquad \text{...(1)}$$

where $\overline{f}_1(s) = \frac{s}{s^2+3^2}$, $\overline{f}_2(s) = \frac{1}{s^2+5^2}$

$$\therefore \quad L^{-1}\left[\overline{f}_1(s)\right] = L^{-1}\left(\frac{s}{s^2+3^2}\right) \& L^{-1}\left[\overline{f}_2(s)\right] = L^{-1}\left(\frac{1}{s^2+5^2}\right) = \frac{1}{5}L^{-1}\left(\frac{5}{s^2+5^2}\right)$$

$$\therefore \quad f_1(t) = \cos(3t) \qquad \text{...(2)}$$

$$\& \quad f_2(t) = \frac{1}{5}\sin(5t) \qquad \text{...(3)}$$

From (1), (2) & (3)

$$I = \int_{u=0}^{t} \cos(3u).\frac{1}{5}\sin\left[5\left(t-u\right)\right]du$$

$$= \frac{1}{5}\int_{u=0}^{t}\frac{1}{2}\left[\sin\left(3u+5t-5u\right)\right] - \sin\left[3u-5t+5u\right]du$$

$$= \frac{1}{10}\int_{u=0}^{t}\left[\sin\left(-2u+5t\right) - \sin\left(su-5t\right)\right]du$$

$$= \frac{1}{10}\left[\frac{-\cos\left(-2u+5t\right)}{-2} + \frac{\cos\left(8u-5t\right)}{8}\right]_{u=0}^{t}$$

$$= \frac{1}{10}\left[\frac{1}{2}\left[\cos\left(3t\right) - \cos 5t\right] + \frac{1}{8}\left[\cos 3t - \cos\left(-5t\right)\right]\right]$$

$$= \frac{1}{10}\left[\frac{1}{2}\left[\cos(3t) - \cos(5t)\right] + \frac{1}{8}\left[\cos 3t - \cos(5t)\right]\right]$$

$$= \frac{1}{10}\left(\cos(3t) - \cos(5t)\left(\frac{4+1}{8}\right)\right) = \frac{1}{10}\frac{5}{8}\left[\cos(3t) - \cos 5t\right]$$

$$= \frac{1}{16}\left[\cos(3t) - \cos(5t)\right]$$

$$\therefore \quad I = L^{-1}\left[\frac{s}{(s^2+3^2)(s^2+5^2)}\right] = \frac{1}{16}\left[\cos(3t) - \cos(5t)\right]$$

Q.5(b) What is the chance of throwing ten with four dice? [6]

Ans.: There are four dice

$\Rightarrow$ n(S) = ${}^6C_1 \cdot {}^6C_1 \cdot {}^6C_1 \cdot {}^6C_1$ = 1296 n(S) = 1296

A : event that sum of the numbers is 10

Now sum = 10, is possible by the following ways

1) (1, 2, 3, 4) these numbers can be arranged in 4! = 24 ways

$\therefore$ (1, 2, 3, 4) $\Rightarrow$ 24 ways

<u>If number is repeated</u>

2) (1, 1, 2, 6) $\Rightarrow \frac{4!}{2!}$ = 12 ways (1, 1, 3, 5) $\Rightarrow \frac{4!}{2!}$ = 12 ways (1, 1, 4, 4) $\Rightarrow \frac{4!}{2!2!}$ = 6 ways	3) If 2 is repeated (2, 2, 1, 5) $\Rightarrow \frac{4!}{2!}$ = 12 ways (2, 2, 3, 3) $\Rightarrow \frac{4!}{2!2!}$ = 6 ways
4) If 3 is repeated (3, 3, 2, 2) already taken	5) If repeated three times (2, 2, 2, 4) $\Rightarrow \frac{4!}{3!}$ = 4 ways (3, 3, 3, 1) $\Rightarrow \frac{4!}{3!}$ = 4 ways

$\therefore$ n(A) = 24 + 12 + 12 + 6 + 12 + 6 + 4 + 4 $\Rightarrow$ n(A) = 80

$\therefore$ P(A) = $\frac{90}{1296}$ $\Rightarrow$ P(A) = 0.062

Q.5(c) In a certain examination there are multiple choice questions. There are four possible answers to each questions and one of them is correct. An intelligent student can solve 90% questions correctly by reasoning and for the remaining 10% questions he give answer by guessing. A week student can solve 20% question correctly by reasoning and for the remaining 80% questions he gives answer by guessing. An intelligent student gets the correct answer. What is the probability that he was guessing. [8]

Ans.: Let c be the event of answer is collect answer.

Let R be the event of answer with reasoning

Let G be the event of answer with questing

By given P(R) = 90% = 0.9, P(C/R) = 1

P(G) = 10% = 0.1, P(C/G) = $\frac{1}{4}$

$$P(c) = P(R).P(C/R).P(G)P(C/G)$$

$$= (0.9\times 1) + \left(0.1\times\frac{1}{4}\right) \qquad = 0.925$$

$$\therefore \quad P(G/C) = \frac{p(G\cap C)}{p(C)} = \frac{p(G)p(C/G)}{p(C)} = \frac{0.1\times\frac{1}{4}}{0.925}$$

$$\therefore \quad P(G/C) = \frac{1}{37} \qquad = 0.027027$$

Q.6(a) A can hit a target 2 times in 5 shots, B 3 times in 4 shots, C 2 times in 3 shots. They fire a volley. What is the probability that at least 2 shots hit the target? [6]

Ans.: Let A be the event that A will hit the target
Let B be the event that B will hit the target
Let C be the event that C will hit the target

$$p(A) = \frac{2}{5},\ p(B) = \frac{3}{4},\ p(C) = \frac{2}{3},\ p(\bar{A}) = \frac{3}{5},\ p(B) = \frac{1}{4},\ p(\bar{C}) = \frac{1}{3}$$

p(at least two hit the target)

$$= p\left[(A\cap B\cap C)\cup(A\cap B\cap\bar{C})\cup(A\cap\bar{B}\cap C)\cup(\bar{A}\cap B\cap C)\right]$$

$$= p(A\cap B\cap C) + p(A\cap B\cap\bar{C}) + p(A\cap\bar{B}\cap C) + p(\bar{A}\cap B\cap C)$$

$$= [p(A)\cdot p(B)p(C)] + [p(A)\cdot p(B)\cdot p(\bar{C})] + [p(A)\cdot p(\bar{B})\,p(C)] + [p[\bar{A}]p(B)p(C)]$$

$$= \left(\frac{2}{5}\times\frac{2}{4}\times\frac{2}{3}\right) + \left(\frac{2}{5}\times\frac{3}{4}\times\frac{1}{3}\right) + \left(\frac{2}{5}\times\frac{1}{4}\times\frac{2}{3}\right) + \left(\frac{3}{5}\times\frac{2}{5}\times\frac{2}{3}\right)$$

$$= \frac{1}{5} + \frac{1}{10} + \frac{1}{15} + \frac{3}{10} = \frac{2}{5}$$

Q.6(b) Find $L^{-1}\left(\tan^{-1}\left(\frac{2}{s^2}\right)\right)$ [6]

Ans.: We have $L^{-1}\ \{\phi(s)\} = \frac{-1}{t}L^{-1}\{\phi^1(s)\}$

$$\therefore \quad L^{-1}\left[\tan^{-1}\left(\frac{2}{s^2}\right)\right] = \frac{-1}{t}L^{-1}\left[\frac{d}{ds}\left(\tan^{-1}\left(\frac{2}{s^2}\right)\right)\right]$$

$$L^{-1}\left[\tan^{-1}\left(\frac{2}{s^2}\right)\right] = \frac{-1}{t}L^{-1}\left[\frac{1}{1+4/s^4}\left(\frac{-4}{s^3}\right)\right] = \frac{-1}{t}L^{-1}\left[\frac{-4s}{s^4+4}\right] = \frac{4}{t}L^{-1}\left[\frac{s}{s^4+4}\right]$$

$$= \frac{4}{t}L^{-1}\left[\frac{s}{(s^2+2)^2-(2s)^2}\right] = \frac{4}{t}.\frac{1}{4}L^{-1}\left[\frac{1}{(s^2-2s+2)} - \frac{1}{(s^2+2s+2)}\right]$$

$$= \frac{1}{t}L^{-1}\left[\frac{1}{(s-1)^2+1} - \frac{1}{(s+1)^2+1}\right] = \frac{1}{t}\left[e^t\sin t - e^{-t}\sin t\right]$$

$$= \frac{2\sin t}{t}\left(\frac{e^t-e^{-t}}{2}\right) = \frac{2\sin t\sinh t}{t}$$

Q.6(c) If R is the relation on the set of integers such that aRb if and only if 2a + 3b is divisible by 5. Find the equivalence classes. [8]

Ans.: By given a R b if and only if 2a + 3b is divisible by 5

we know 2a + 3a = 5a is divisible

$\therefore$ $\Rightarrow$ a Ra $\therefore$ R is reflexive

Let a Rb $\Rightarrow$ 2a + 3b is divisible 5

$\Rightarrow$ 2a + 3b = 5k ...(1)

where k is an integer

slow 2b + 3a = $(5b - 3b) + (5a - 2a)$

= (5b + 5a) – 2a – 3b

= 5(b + a) – (2a +3b)

on using (1)

Vs + 3a = 5(a + b) – 5k = 5(a + b – k) is divisible by 5

$\Rightarrow$ 2b + 3a is divisible by 5

$\Rightarrow$ b Ra

Thus a Rb $\Rightarrow$ b Ra $\therefore$ R is symmetric

Now let a Rb and bRc

$\therefore$ Those exist integers k_1 & k_2 such that

2a + 3b = $5k_1$ ...(2)

& 2b + 3c = $5k_2$ ...(3)

(2) + (3)

(2a + 3b) + (2b + 3c) = $5k_1 + 5k_2$

(2a + 3c) + 2b = $5(k_1 + k_2)$

$\therefore$ 2a + 3c = $5(k_1 + k_2) - 5b$

$\therefore$ 2a + 3c = $5[k_1 + k_2 - b]$ is divisible by 5

$\Rightarrow$ a Rc

Thus aRb & bRc $\Rightarrow$ aRc

$\therefore$ r is transitive, Thus R is reflexive, symmetric and transitive R in an equivalence relation.

To find equivalence classes

[a] = $\{x \in 2 / arx \text{ ie } 2a + 3x = 5k, k4 \text{ int eger}\}$

[0] = {..., –15, –10, –5, 0, 5, 10, 15, ...}

[1] = {..., –14, –9, –4, 1, 6, 11, 16, ...}

[2] = {..., –13, –8, –3, 2, 7, 12, 17, ...}

[3] = {..., –12, –7, –2, 3, 8, 13, 18, ...}

[4] = {–11, –6, –1, 4, 9, 14, 19, ...}

$\therefore$ the partition of z induced by R

i.e. [z/R] = {[0], [1], [2], [3], [4]}

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