

**Prelim Paper**

Time: 3 Hrs.]

**Applied Mathematics - III**

[Marks : 80

- N.B.:** (1) Question No. 1 is compulsory.  
 (2) Attempt any **THREE** of the remaining.

1. (a) Find Laplace transform of  $te^{3t} \cos t$ . [5]  
 (b) Evaluate  $\oint_c \frac{z-1}{z^2+2z+5} dz$ , where  $c$  is  $|z+1+i|=2$ . [5]  
 (c) Show that  $f(z) = \sinh z$  is analytic. Hence find its derivative. [5]  
 (d) Compute spearman's rank correlation for the data : [5]

X :	18	20	34	52	12
Y :	39	23	35	18	46

2. (a) Show that the function  $w = \frac{4}{z}$  transforms the straight line  $x = c$  in the  $z$ -plane into circle in  $w$ -plane. Find its centre and radius. [6]  
 (b) Show that  $\int_0^\infty e^{-t} \int_0^t \frac{\sin u}{u} du dt = \frac{\pi}{4}$  [6]  
 (c) Obtain fourier series for  $f(x) = \begin{cases} 1 + \frac{2x}{\pi} & -\pi \leq x \leq 0 \\ 1 - \frac{2x}{\pi} & 0 \leq x \leq \pi \end{cases}$  [8]

Hence deduce that  $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$

3. (a) Evaluate  $\oint_c \frac{e^{kz}}{z} dz$ , where  $c : |z|=1$ . Hence deduce that  $\int_0^\pi e^{k \sin \theta} \cos(k \sin \theta) d\theta = \pi$  [6]  
 (b) For the lines of regression  $6y - 5x = 90$ ,  $15x - 8y = 130$  and  $\sigma_x^2 = 16$  Find (i)  $\bar{x}, \bar{y}$  (ii)  $r$  (iii)  $\sigma_y$  [6]  
 (c) Solve the differential equation  $\frac{dy}{dx} + 2y + \int_0^t y dt = \sin t$  using Laplace transform [8]  
 give  $y(0) = 1$

4. (a) Find Laurentz's series for  $f(z) = \frac{2}{(z-1)(z-2)}$  indicating region of convergence. [6]  
 (b) Show that  $\cos x = \frac{8}{\pi} \sum_{m=1}^\infty \frac{m}{4m^2-1} \sin(2mx)$  If  $0 < x < \pi$  [6]  
 (c) Find Bilinear transformation which maps the points  $1, i, -1$  onto the points  $i, 0, -i$ . Hence find fixed points and image of  $|z| < 1$ . [8]

5. (a) Solve using Bender-Schmidt method  $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$ , subject to the conditions [6]

$$u(0, t) = 0, u(1, t) = 0, u(x, 0) = \sin \pi x, 0 \leq x \leq 1$$

- (b) Determine the solution of one-dimensional heat equation under the boundary [6]  
conditions,  $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$

$$u(0, t) = 0, u(\ell, t) = 0, u(x, 0) = x, (0 < x < \ell), \ell \text{ being length of the rod.}$$

- (c) Find inverse Laplace Transform of [8]

(i)  $\log\left(1 + \frac{\alpha^2}{s^2}\right)$                       (ii)  $\frac{e^{-s}}{s^2 + s + 1}$

6. (a) Obtain complex form of fourier series for  $F(x) = e^{ax}$ , in  $(-\pi, \pi)$  where  $a$  is not an integer. [6]

- (b) Fit a curve  $y = a \cdot b^x$  to the following data, using method of least squares. [6]

X :	2	3	4	5	6
Y :	144	172.8	207.4	248.8	298.5

- (c) (i) Evaluate  $\int_0^{2\pi} \frac{d\theta}{5 + 3 \sin \theta}$  using Residue Theorem. [4]

- (ii) Evaluate  $\int_{-\infty}^{\infty} \frac{dx}{x^2 + 1}$  using Residue Theorem. [4]

