Strength of Materials

Time : 3 Hrs.] Prelim Paper Solution [Marks : 80

Q.1 Answer any FOUR of the following: [20]

Q.1(a) Derive formula for elongation of bar due to its self weight. [5]

Ans.: Elongation of bar due to its own weight :

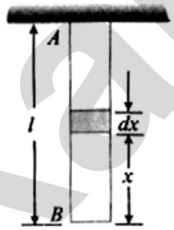
Consider a bar AB hanging freely under its own weight as shown in figure.

Let l = Length of the bar

A = Cross-sectional area of the bar

E = Young's modulus for the bar material

and w = Specific weight of the bar material



Now consider a small section dx of the bar at a distance x from B.

We know that weight of the bar for a length of x,

$$P = wAx$$

$\therefore$ Elongation of the small section of the bar, due to weight of the bar for a small section of length x,

$$= \frac{Pl}{AE} = \frac{(wAx)\,dx}{AE} = \frac{wx\,dx}{E}$$

Total elongation of the bar may be found out by integrating the above equation between zero and l. Therefore total elongation,

$$\delta l = \int_0^l \frac{wx\,dx}{E} = \frac{w}{E}\int_0^l x\,dx = \frac{w}{E}\left[\frac{x^2}{2}\right]_0^l$$

or $$\delta l = \frac{wl^2}{2E} = \frac{Wl}{2AE} \quad \ldots (\because\ W = wAl = \text{Total weight})$$

Note : From the above result, we find that the deformation of the bar, due its own weight, is equal to half of the deformation, if the same body is subjected to a direct load equal to the weight of the body.

Q.1(b) A circular alloy bar 2m long uniformly tapers from 30 mm diameter to 20 mm diameter. Calculate the elongation of the rod under an oxial force of 50 kN. Take 'E' for the alloys as 140 GPa. [5]

Ans.: Given : Length of bar (ℓ) = 2 m = 2×10^3 mm ; Diameter of section 1(d_1) =30mm; Diameter of section 2(d_2) = 20 mm; Axial force (P) = 50kN = 50×10^3N and modulus of elasticity (E) = 140 GPa = 140×10^3 N/mm^2

We know that elongation of the rod,

$$\delta\ell = \frac{4P\ell}{\pi E d_1 d_2}$$

$$= \frac{4 \times (50 \times 10^3) \times (2 \times 10^3)}{\pi \times (140 \times 10^3) \times 30 \times 20}$$

$$= 1.52\text{mm}$$

Q.1(c) State the assumption in the theory of pure bending and derive the formula. [5]

$$\frac{M}{I} = \frac{\sigma}{Y} = \frac{E}{R}$$

Ans.: The following are the assumptions made in theory of bending :

1. The material of beam is homogeneous and isotropic.
2. The beam is straight before loading.
3. The beam is of uniform cross section through its length.
4. Transverse section, which are plane before loading remains plane even after loading.
5. The material is elastic and Hook's law is obeyed.
6. The effect of shear is neglected. Therefor the analysis is meant for pure bending.
7. The modulus of elasticity (E) has the same value in tension and compression.
8. Each layer is free to expand or contract having no influence in the neighboring layers for their extension or contraction.
9. The beam is initially straight and all longitudinal filaments bent into circular arcs with a common center of curvature.

Derivation of Flexural Formula (Equation of Bending)

Consider a slice ABCD of small length dx of a simply supported beam subjected to bending moment 'M' as shown in Figure (a). EF is the neutral axis. It is obvious that AB = CD = EF = dx (before bending). A bent up beam is as shown in Figure (d). The new lengths of AB and CD are A_1B_1 and C_1D_1 respectively. For a neutral layer EF, E_1F_1 = EF = dx. The bent up curved portion is assumed as small arc of the circle, with O as centre. Let θ be the angle subtended at the centre by the arc. Let R be the radius of curvature of the bent up beam. The neutral axis E_1F_1 lies at a radius R of the circle.

Length of the arc $E_1F_1 = R\,\theta$

Now, consider a layer GH at a distance 'y' from the neutral axis EF. Let this layer be compressed to G_1H_1 after bending.

Since there is no change in the length of the neutral layer, we have

$$E_1\,F_1 = E\,F = GH = R\,.\,\theta \qquad \text{...(i)}$$

Naturally, length of the $G_1H_1 = (R - y)\,\theta$ ...(ii)

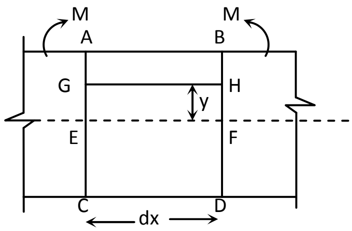
Now, decrease in the length of layer GH is given by,

$$\delta L = \text{Initial length} - \text{Final length} = GH - G_1H_1$$

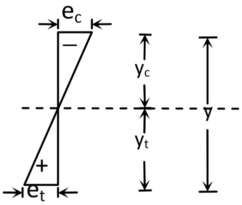
$$= R\theta - (R - y)\theta = y.\theta$$

$\therefore$ Strain in the layer GH is given by,

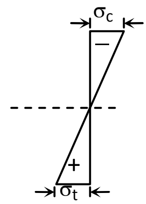
$$e = \frac{\delta L}{L} = \frac{y.\theta}{R\theta} = \frac{y}{R} \qquad \text{...(iii)}$$



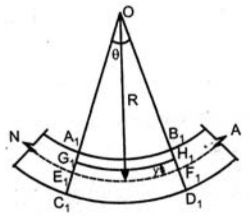
(a) Beam before loading



(b) Strain distribution diagram



(c) Stress distribution diagram



(d) Beam after bending

But $\qquad \text{Strain} = \dfrac{\text{Stress}}{\text{Young's modulus}}$

i.e. $\qquad e = \dfrac{\sigma}{E}$

Substituting the value of 'e' in the equation (iii), we have

$$\frac{\sigma}{E} = \frac{y}{R}$$

$\therefore \qquad \dfrac{\sigma}{y} = \dfrac{E}{R} \qquad$ ...(A)

Now, from the equation (iii), we have

$$e \propto y$$

i.e. the strain of any layer is proportional to its distance from the N.A.

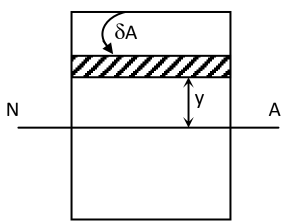
At N.A., $\quad y = 0$

$\therefore$ Strain at N.A. is always zero.

All the layer above the N.A. are compressed and those below the N.A. are elongated. The maximum compressive strain (e_c) occurs at the topmost layer A_1B_1 and the maximum tensile strain occurs at the bottommost layer C_1D_1. The bending strain distribution diagram is as shown in Figure (b).

[Since compression causes decrease in the length and tension cause increase in the length of the layer, compressive strain is generally taken as negative and tensile strain as positive.]

Now, consider a small area 'δA' at a distance 'y' from the neutral axis as shown in Figure (e).



(e)

From the equation (i),

Stress on this small area, $\sigma = \frac{E}{R}\times y$

$\therefore$ Small force on this area

$$\delta F = \text{Stress} \times \text{Area}$$

$$= \sigma \times \delta A = \left(\frac{E}{R}\times y\right)\times \delta A$$

Moment of this small force about N.A.

$$\delta M = \delta F \times y = \left(\frac{E}{R}\times y\right)\delta A \times y = \frac{E}{R}\times \delta A \times y^2$$

$$\therefore \text{ Total moment, } M = \sum \delta M = \sum \frac{E}{R}\times \delta A \times y^2 = \frac{E}{R}\sum \delta A \,.\, y^2$$

$$= \frac{E}{R}\times I \quad \ldots\left(\because \sum \delta A \,.\, y^2 = I\right)$$

$$\therefore \quad \frac{M}{I} = \frac{E}{R} \quad \ldots(B)$$

Combining the two equations (A) and (B), we have

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

Q.1(d) A hollow circular shaft of 80 mm internal diameter and 150 mm external diameter is subjected to a torque of 70 kN-m. Find maximum shear stress developed. **[5]**

Ans.: Given : $T = 70$ kN-m $= 70 \times 10^6$ N-mm
External diameter of shaft (D) = 150 mm
Internal diameter of shaft (d) = 80 mm
Shear stress at outer radius i.e. at R = 75 mm

Using torsional equation,

$$\therefore \quad \frac{\tau}{R} = \frac{T}{J}$$

$$J = \frac{\pi}{32}(D^4 - d^4) = \frac{\pi}{32}(150^4 - 80^4) = 45.68\times 10^6 \text{ mm}^4$$

$$\therefore \quad \frac{\tau}{75} = \frac{70\times 10^6}{45.68\times 10^6}$$

$$\tau = 114.93 \text{ N/mm}^2$$

Shear stress at inner radius i.e at r = 40 mm.

Using torsional equation :

$$\therefore \quad \frac{\tau}{R} = \frac{T}{J}$$

$$\frac{\tau}{40} = \frac{70\times 10^6}{45.68\times 10^6}$$

$$\tau = 61.30 \text{ N/mm}^2$$

Maximum value of shear stress is developed at outer section,

$$\tau = 114.93 \text{ N/mm}^2$$

Q.1(e) Calculate the strain energy stored in a bar 2m long, 50 mm wide and 40 mm thick when it is subjected to a tensile load of 60 kN. Take 'E' as 200 GPa. **[5]**

Ans.: Given : Length of bar (ℓ) = 2 m = 2×10^3 mm ; Width of bar (b) = 50mm;

Thickness of bar (t)=40mm ; Tensile load on bar (P) = 60kN = 60×10^3N and modulus of elasticity (E) = 200 GPa = 200×10^3 N/mm^2

We know that stress in the bar,

$$\sigma = \frac{P}{A} = \frac{60\times10^3}{50\times40} = 30\text{N /mm}^2$$

$\therefore$ Strain energy stored in the bar,

$$U = \frac{\sigma^2}{2E}\times V = \frac{(30)^2}{2\times(200\times10^3)}\times4\times10^6 \text{ N}-\text{mm}$$

$= 9 \times 10^3$ N–mm = **9kN–mm**

Q.1(f) A cantilever bean 4m long carries a gradually varying load, zero at the free end to 3 kN/m at the fixed end. Draw B.M. and S.F. diagrams for the beam. **[5]**

Ans.: Given : Span (ℓ) = 4m and gradually varying load at A(w) = 3kN/m. The cantilever beam is shown Fig. (a).

Shear Force diagram

The shear force diagram is shown in Fig. (b) and the values are tabulated here :

$F_B = 0$

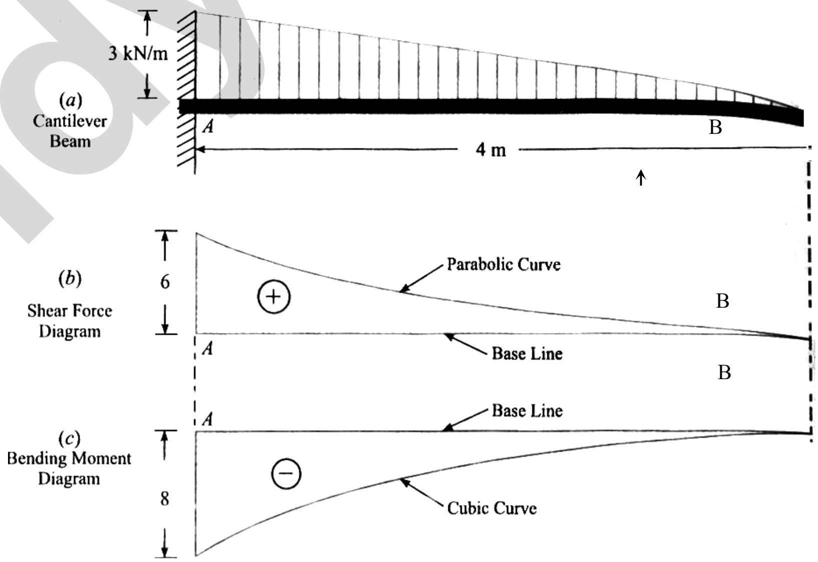
$$F_A = +\frac{3\times4}{2} = +6 \text{ kN}$$

Bending moment diagram

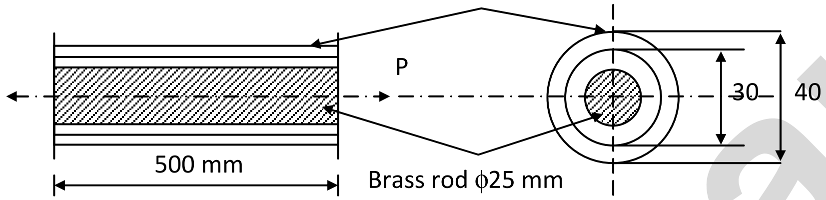
The bending moment diagram is shown in fig. (c) and the values are tabulated here :

$M_B = 0$

$$M_A = -\frac{3\times(4)^2}{6} = -8\text{kN}-\text{m}$$



Q.2(a) A composite bar is made up of a brass rod of 25 mm diameter enclosed in a steel tube of 40 mm external diameter and 30 mm internal diameter as shown in figure. The rod and tube, being coaxial and equal in length, are securely fixed at each end. If the stresses in brass and steel one not to exceed 70 MPa and 120 MPa respectively. Find the load (P) the composite bar can safely carry. **[10]**



Ans.: Given :

Diameter of brass rod = 25mm;
External diameter of steel tube = 40mm;
Internal diameter of steel tube = 30mm;
Maximum stress in brass ($\sigma_{B(max)}$) = 70MPa = 70N/mm^2;
Maximum stress in steel ($\sigma_{S(max)}$) = 120MPa = 120N/mm^2 ;
Length of brass rod (ℓ_B) = ℓ_S = 500mm;
Modulus of elasticity of steel $(E_S)_2$ = 200GPa = 200×10^3 N/mm^2 and
modulus of elasticity of brass (E_B) = 80 GPa = 80×10^3 N/mm^2
Load the composite bar can safely carry

We know that area of brass rod, $A_B = \frac{\pi}{4} \times (25)^2 = 491\text{mm}^2$

and area of steel tube, $A_S = \frac{\pi}{4} \times [(40)^2 - (30)^2] = 550\text{mm}^2$

We also know that as the brass rod and steel tube are securely fixed at each end, therefore strains in both of them will be equal i.e.

$$EB = ES \quad \text{or} \quad \frac{\sigma_B}{E_B} = \frac{\sigma_S}{E_S}$$

First of all, let us find out the maximum stresses in the brass rod and steel tube. We know that when stress in the brass is 70N/mm^2 (maximum permissible), then stress in the steel tube.

$$\sigma_S = \frac{E_S}{E_B} \times \sigma_B = \frac{200}{80} \times 70 = 175 \text{ N/mm}^2$$

It is more than permissible stress in the steel (which is given as 120N/mm^2). Therefore we cannot accept these values of stresses in brass and steel. Now when the stress in steel tube is 120N/mm^2 (Maximum permissible), then stress in the brass rod,

$$\sigma_B = \frac{E_B}{E_S} \times \sigma_S = \frac{80}{200} \times 120 = 48 \text{ N/mm}^2$$

It is less than the permissible stress in brass (which is given as 70N/mm^2). Thus we shall take the stresses in the brass rod (σ_B) and steel tube (σ_S) as 48 N/mm^2 and 120N/mm^2 respectively. Therefore load which the composite bar can carry.

$$P = (\sigma_B.A_B) + (\sigma_S.A_S) = (48 \times 491) + (120 \times 550) \text{ N}$$
$$= 89570 \text{ N} = 89.57\text{kN} \quad \text{Ans.}$$

Change in length : We also know that change in length in the composite bar.

$$\delta\ell = \frac{\sigma.\ell}{E} = \frac{\sigma_B \times \ell_B}{E_B} = \frac{48 \times 500}{80 \times 10^3} = 0.3\text{mm}$$

Note : The change in length of the composite bar may also be found out by the stress in steel from the relation :

$$\delta\ell = \frac{\sigma_S \times \ell_S}{E_S} = \frac{120 \times 500}{200 \times 10^3} = 0.3\text{mm}$$

Q.2(b) A timber beam of rectangular section has a span of 4.8 meters and is simply supported at it ends. It is required to carry a total load of 45 kN uniformly distributed over the whole span. Find the values of the breadth (b) and depth (d) of the beam, if maximum bending stress is not to exceed 7 MPa and maximum deflection is limited to 9.5 mm. Take E for timber as 10.5 GPa. **[10]**

Ans.: Given : Span (ℓ) = 4.8m = 4.8×10^3 mm; Total load (W) = ($w\ell$) = 45kN = 45×10^3 N; Maximum bending stress σb(max) = 7MPa = 7/mm^2; Maximum deflection (y_C) = 9.5mm and modulus of elasticity (E) = 10.5 GPa = 10.5×10^3N/mm^2.

Let b = Breadth of the beam and

d = Depth of the beam

We know that in a simply supported beam, carrying a uniformly distributed load, the maximum bending moment.

$$M = \frac{w\ell^2}{8} = \frac{w\ell \times \ell}{8} = \frac{w \times \ell}{8} = \frac{45 \times 4.8}{8} = 27\,\text{kN}-\text{m} = 27 \times 10^{-6} \text{ N-mm}$$

And moment of inertia of a rectangular section,

$$I = \frac{bd^3}{12}$$

We also know that distance between the neutral axis of the section and extreme fibre,

$$y = \frac{d}{2}$$

∴ Maximum bending stress [$\sigma_{b(max)}$],

$$7 = \frac{M}{I} \times y = \frac{27 \times 10^6}{\frac{bd^3}{12}} \times \frac{d}{2} = \frac{162 \times 10^6}{bd^2}$$

Or $bd^2 = \dfrac{162 \times 10^6}{7} = 23.14 \times 10^6$

We know that maximum deflection (y_C),

$$9.5 = \frac{5w\ell^4}{384E\ell} = \frac{5(45 \times 10^3) \times (4.8 \times 10^3)^3}{384 \times (10.5 \times 10^3) \times \frac{bd^3}{12}} = \frac{74.1 \times 10^9}{bd^3}$$

$$\therefore \quad bd^3 = \frac{74.1 \times 10^9}{9.5} = 7.8 \times 10^9$$

Driving equation (ii) by equation (i),

$$d = \frac{7.8 \times 10^9}{23.14 \times 10^6} = \mathbf{337mm}$$

Substituting this value of d in equation (i) ,

$$b \times (337)^2 = 23.14 \times 106$$

$$b = \frac{23.14 \times 10^6}{(337)^2} = \mathbf{204\ mm}$$

Q.3(a) A plane element in a boiler is subjected to tensile stresses of 400 MPa on one plane and 150 MPa on the other at right angles to the former. Each of the above stresses is accompanied by a shear stress of 100 MPa such that when associated with the minor tensile stress tends to rotate the element in anticlockwise direction. Find **[10]**

(i) Principal stresses and their directions

(ii) Maximum shearing stresses and the direction of the plane on which they act.

Ans.: Given : Tensile stress along x–x axis (σ_x) = 400MPa ;

Tensile stress along y–y axis (σ_y) = 150MPa and

shear stress (τ_{xy}) =100MPa (Minus sign due to anticlockwise on x–x direction).

(i) Principal stresses and their directions

We know that maximum principal stress

$$\sigma_{max} = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{400 + 150}{2} + \sqrt{\left(\frac{400 - 150}{2}\right)^2 + (-100)^2}\ \text{MPa}$$

$$= 275 + 160.1 = 435.1\ \text{MPa}$$

And minimum principal stress,

$$\sigma_{min} = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{400 + 150}{2} - \sqrt{\left(\frac{400 - 150}{2}\right)^2 + (-100)^2}\ \text{MPa}$$

$$\mathbf{= 275 - 160.1 = 114.9MPa}$$

Let θp = Angle which plane of principal stress makes with x–x axis.

We know that,

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2 \times 100}{400 - 150} = 0.8 \text{ or } 2\theta_p = 38.66^\circ$$

$$\theta_p = 19.33^\circ \quad \text{or } 109.33^\circ$$

(ii) Maximum shearing stresses and their directions

We also know that maximum shearing stress

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{400 - 150}{2}\right)^2 + (-100)^2}$$

$$= 160.01\ \text{MPa}$$

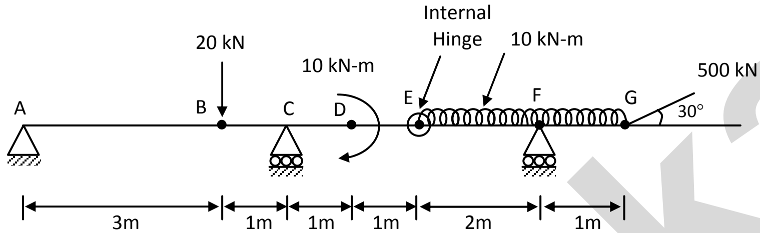
Let θ_x = Angle which plane of maximum shearing stress makes with x–x axis.

We know that,

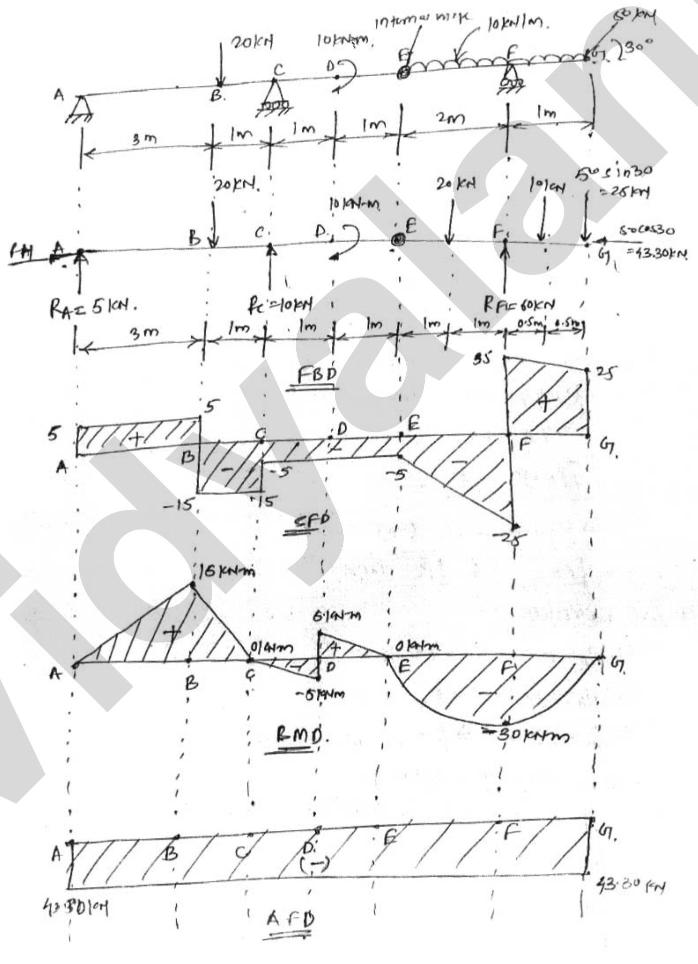
$$\tan 2\theta_x = \frac{\sigma_x - \sigma_y}{2\,\tau_{xy}} = \frac{400-150}{2\times 100} = 1.25 \quad \text{or} \quad 2\theta_s = 51.34^\circ$$

$$\boldsymbol{\theta_S = 25.67^\circ \text{ or } 115.67^\circ}$$

Q.3(b) Draw the axial force, shear force and bending moment diagrams for the beam loaded as below : [10]



Ans.:



Support reaction calculations:

Applying condition of equilibrium to member GE alone,

$\sum M_E = 0$ [+↻ ↺–]

$(25 \times 3) - (R_F \times 2) + (30 \times 1.5) = 0$

$2R_F = 120$ $R_F = 60$ kN

Applying condition of equilibrium to A_E $\sum M_E = 0$ +↻ ↺–

$6R_A + 2R_C + 10 - 20\,(3) = 0$

$6R_A + 2R_C = 60 - 10 = 50$

$6R_A + 2R_C = 50$ ...(1)

Condition of equilibrium for entire beam $\sum F_y = 0$ ↑ (+) ↓ (–)

$R_A + R_C + R_F - 20 - 30 - 25 = 0$

$R_A + R_C = 20 + 30 + 25 - 60$ ...[$R_F = 60$]

$R_A + R_C = 15$ ...(2)

From equation (1) and (2) we get

$R_A = 5$ kN $R_C = 10$ kN

S.F. calculation:

$SF_G|_R = 0 SF_G|_L = 25$ kN

$SF_H|_R = 25$ kN $SF_H|_L = 35$ kN

$SF_F|_R = 35$ kN $SF_A|_L = 35 - 60 = -25$ kN

$SF_E = -25 + 20 = -5$ kN

SF from section E to D is constant = –5 kN

$SF_C|_R = -5$ kN $SF_C|_L = -5 - 10 = -15$ kN

$SF_B|_R = -15$ kN $SF_B|_L = -15 + 20 = 51$ kN

$SF_A|_R = 5$kN $SF_A|_L = 5 - 5 = 0$ kN

B.M. calculations:

$BM|_G = 0$ kN-m

$BM|_F = (-25 \times 1) - (10 \times 0.5) = -25 - 5 = -30$ kN–m

$BM|_E = (-25 \times 3) - (10 \times 2.5) + (60 \times 2) - (20 \times 1) = 0$ kN–m

$BM_D|_R = (-25 \times 4) - (10 \times 3.5) + (60 \times 3) - (20 \times 2) = 5$ kN–m

$BM_D|_L = 5 - 10 = -5$ kN–m

$BM|_C = (-25 \times 5) - (10 \times 4.5) + (60 \times 4) - (20 \times 3) - 10 = 0$ kN–m

$BM|_B = (-25 \times 6) - (10 \times 5.5) + (60 \times 5) - (20 \times 4) - 10 + (10 \times 1) = 15$ kN–m

$BM|_A = 0$ kN–m

Axial force calculation:

$\sum F_x = 0$ $\xrightarrow{+}$ $\xleftarrow{-}$

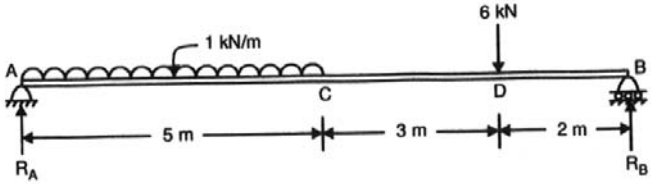
$R_A - 43.30 = 0$ $R_A = 43.30$ kN

$SF_F|_R = 0$ kN $SF_F|_L = -43.30$ kN

$SF_A|_R = -43.30$ kN $SF_A|_L = -43.30 + 43.30 = 0$kN

S.F., B.M., A.F. diagram are as shown in figure.

Q.4(a) **For the beam loaded as shown in the figure, find slopes at A and B and deflection at C and D. Also find the position and value of maximum deflection in the beam. Take E = 2 $\times$ 10^5 N/mm^2, I = 1 $\times$ 10^8 mm^4.** **[10]**



Ans.: Support Reaction Calculation

$\sum M_A = 0 \circlearrowleft + ve$

$-(1 \times 5) \times 2.5 - 6 \times 8 + R_B \times 10 = 0$

$\therefore \quad R_B = 6.05 \text{ kN} \uparrow$

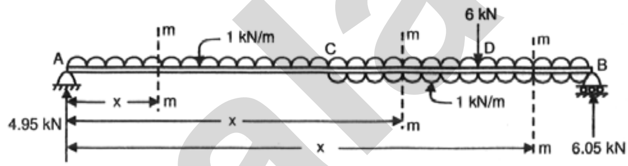
$\sum F_y = 0 \quad \uparrow + ve$

$R_A - 5 - 6 + 6.5 = 0$

$\therefore \quad R_A = 4.95 \text{ kN} \uparrow$

Using Macaulay's Method :

Taking sections m-m at a distance x from end A. The continuous moment equation valid for entire beam is written by taking the section m-m whenever the loading changes. i.e. in portions AC, CD and DB of the beam.



$$EI\frac{d^2y}{dx^2} = M \qquad \text{(Taking sagging moments as positive)}$$

$$EI\frac{d^2y}{dx^2} = 4.95x - 1\times\frac{x^2}{2}\Bigg|_0^5 + 1\times\frac{(x-5)^2}{2}\Bigg|_0^8 - 6(x-8)\Big|_0^{10}$$

Slope Equation

$$EI\frac{dy}{dx} = C_1 + \frac{4.95\,x^2}{2} - \frac{x^3}{6}\Bigg|_0^5 + \frac{(x-5)^3}{6}\Bigg|_0^8 \frac{-6(x-8)^2}{2}\Bigg|_0^{10}$$

Deflection Equation

$$EI.y = C_1x + C_2 + \frac{4.95x^3}{6} - \frac{x^4}{24}\Bigg|_0^5 + \frac{(x-5)^4}{24}\Bigg|_0^8 - \frac{6(x-8)^3}{6}\Bigg|_0^{10}$$

[At x = 0, y = 0]…… because deflection at supports is zero.

$0 = 0 + C_2 + 0 - 0 \qquad \therefore \quad C_2 = 0$

[At x = 10, y = 0]… because deflection at the support is zero

$$\therefore \quad 0 = C_1\times(10) + \frac{4.95(10)^3}{6} - \frac{(10)^4}{24} + \frac{(10-5)^4}{24} - \frac{6(10-8)^3}{6}$$

$C_1 = -42.63$

For maximum deflection, we will have to use trial and error method.

Let us assume maximum deflection is in portion CD

$\therefore$ equating the slope $\frac{dy}{dx}$ in portion CD equal to zero.

$$0 = -42.63 + \frac{4.95x^2}{2} - \frac{x^3}{6} + \frac{(x-5)^3}{6}$$

$$0 = -42.63 + 2.46x^2 - \frac{x^3}{6} + \frac{1}{6}\left(x^3 - 15x^2 + 75x - 125\right)$$

$$0 = -0.04x^2 + 12.5x - 63.46$$

$\therefore$ x = 5.13 m

$\therefore$ Maximum deflection is at x = 5.13 m from end A

putting x = 5.13 m in deflection equation

$$EI.y = -42.63 \times 5.13 + 4.95 \times \frac{5.13^3}{6} - \frac{5.13^4}{24} + \frac{0.13^4}{24}$$

$$EI.y = -136.17\ kNm^3$$

$$EI.y = -136.17 \times 10^{12}\ Nmm^3$$

$$y_{max} = -\frac{136.17 \times 10^{12}}{EI}\ mm$$

$$y_{max} = \frac{136.17 \times 10^{12}}{(2 \times 10^5) \times (1 \times 10^8)} = -6.81\ mm$$

$\therefore$ y_{max} = 6.81 mm ↓ at x = 5.13 m to the right of end A

Q.4(b) A thin cylindrical shell having 120 cm diameter, thickness of metal 15 mm and 4 m long, is subjected to an internal pressure of 2.5 N/mm². Find the Hoop stress, longitudinal stress, changes in length, diameter and volume of the shell. Take E = 2 × 10⁴ kN/cm² and Poisson ratio = 0.3 **[10]**

Ans.: Given : Shell-thin cylinder,

L = 4m = 4000 mm d = 120 cm = 120mm

t = 15 mm P = 2.5 N/mm²

$E = 2 \times 10^4\ kN/cm^2 = 200 \times 10^3\ N/mm^2$

$\mu = \frac{1}{m} = 0.3$

Circumferential/hoop stress, $F_c = \frac{Pd}{2t} = \frac{(2.5)(1200)}{2 \times 15} = 100\ N/mm^2$

Longitudinal stress, $F_L = \frac{Pd}{4t} = \frac{(2.5)(1200)}{4(15)}$

$= 50\ N/mm^2$

Circumferential strain, $e_c = \frac{F_c}{E} - \frac{1}{m}\frac{F_L}{E} = \frac{100}{(200 \times 10^3)} - 0.3\frac{50}{(200 \times 10^3)}$

$= 4.25 \times 10^{-4}$

Also circumferential strain $e_c = \frac{\delta d}{2}$

or $\delta d = e_c \times d = (4.25 \times 10^{-4}) \times 1200$

$\therefore$ $\delta d = 0.51$ mm

Longitudinal strain, $e_L = \frac{F_L}{E} - \frac{1}{m}\frac{F_c}{E} = \frac{50}{(200\times10^3)} - (0.3)\frac{100}{(200\times10^3)}$

$= 1 \times 10^{-4}$

Also longitudinal strain, $e_L = \frac{\delta L}{L}$

$\delta L = e_L L = (1 \text{ x } 10^{-4}) \times 4000$

$= 0.4$ mm

Volumetric strain, $e_v = 2e_c + e_L = (2 \times 4.25 \times 10^{-4}) + (1 \times 10^{-4})$

$= 9.5 \times 10^{-4}$

Now, $e_v = \frac{\delta V}{V}$

$\delta V = e_V \times V$

$V = \frac{\pi}{4} d^2 . L = \frac{\pi}{4}(1200^2)\,(4000)$

$= 4.52 \times 10^9 \text{ mm}^3$

$= (9.5 \times 10.4) \times 4.52 \times 10^9$

$\delta V = 4.298 \times 10^6 \text{ mm}^3$

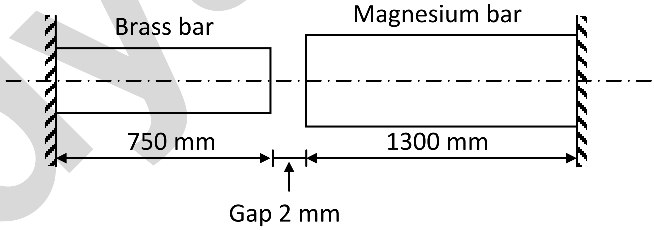
Q.5(a) A bimetallic thermal control shown in figure is made of bass bar of length 750 mm and cross sectional area 100 mm^2 and magnesium bar of length 1300 mm and cross sectional area 200 mm^2. The two bars are arranged so that the gap between their free ends is 2mm at room temperature, calculate : **[10]**

(i) The temperature rise at which the two bars come in contact.

(ii) The stress in the materials when the temperature increase is 300°C.

take, E_b = 150 GPa, E_m = 65 GPa

$\alpha_b = 10 \times 10^{-6}$ / °C, $\alpha_m = 14.5 \times 10^{-6}$ /°C

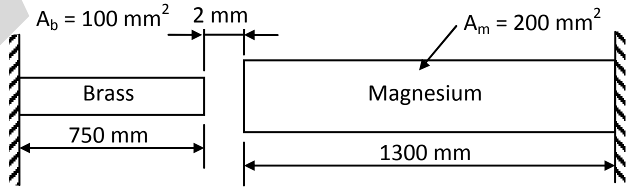


Ans.: E_b = 150 GPa

E_m = 65 GPa

$\alpha_b = 10 \times 10^{-6}$ /° C.

$\alpha_m = 14.5 \times 10^{-6}$/°C



Let 't' be the rise in temperature using suffix 'b' for brass and m for magnesium, we have

$\delta_b = \alpha_b t \cdot \ell_b = (10 \times 10^{-6})\ (750)\ (t)$

$\delta_m = \alpha_m t \cdot \ell_m = (14.5 \times 10^{-6})\ (1300)\ (t)$

Hence, $\delta_b + \delta_m = 2$ mm

$[(10 \times 10^{-6})\ (750)\ t] + [(14.5 \times 10^{-6})\ (1300)\ t] = 2$

$\therefore$ t = 75.90°C

Now, when t = 300°C

$\delta_b = (10 \times 10^{-6})\ (750)\ (300) = 2.25$ mm

$\delta_m = (14.5 \times 10^{-6})\ (1300)\ (300) = 5.655$ mm

Now, expansion restricted,

$\delta = (2.25 + 5.655) - 2 = 5.905$ mm ...(1)

If 'p' is the compressive force induced, compression δ of the two bars is,

$$\delta = \frac{PL_b}{A_bE_b} + \frac{PL_m}{A_mE_m} = P\left[\frac{750}{(100)(150\times10^3)} + \frac{1300}{(200)(65\times10^3)}\right] = (1.5\times10^{-4})P \quad ...(2)$$

Equating (1) and (2) we get,

$(1.5 \times 10^{-4})\ P = 5.905$

P = 39.37 kN

$$\sigma_b = \frac{P}{A_b} = \frac{39.37\times10^3}{100} = 393.667\ \text{N/mm}^2$$

$$\sigma_m = \frac{P}{A_m} = \frac{39.37\times10^3}{200} = 196.85\ \text{N/mm}$$

Q.5(b) Determine the diameter of the shaft to transmit 1 MW rotating at 220 r.p.m. and the marking conditions to be satisfied are : [10]

(i) That the shaft not twist more than 1° on length of 12 diameters and

(ii) The shear stress must not exceed 60 N/mm^2 take G = 84 kN/m^2.

Ans.: Given : Power(P) = 1 MW = 1×10^6 watt

Speed (N) = 220 rpm

Let diameter of shaft = d

$$\theta = 11 \times \frac{\pi}{180} = 0.0175 \text{ rad.}$$

Length or shaft (L) = 12d

$\tau = 60$ N/mm^2

G = 84 kN/mm^2 = 84×10^3 N/mm^2

We have,

$$\text{Power (p)} = \frac{2\pi NT}{60}$$

$$(1 \times 10^6) = \frac{2\pi\times220\times T}{60}$$

$\therefore$ T = 43405.894 N-m = 43405.894×10^3 N/mm

To satisfy twist criteria :

It is repaired that $\theta \le 1° = 0.0175$ rad

$$J = \frac{\pi}{32} d^4$$

$L = 12d$

Using torsion equation,

$$\therefore \quad \frac{T}{J} = \frac{G\theta}{L}$$

$$\frac{(43405.894 \times 10^3)}{\frac{\pi}{32}.d^4} = \frac{(84 \times 10^3)\ (0.0175)}{12d}$$

$d^3 = 3609214.74$

$d = 153.393$ mm

To satisfy shear stress criteria,

It is required that, $\tau \leq 60$ N-mm^2

Using torsion equation,

$$\therefore \quad = \frac{T}{J}$$

$$\frac{60}{\frac{d}{2}} = \frac{43405.894 \times 10^3}{\frac{\pi}{32}.d^4}$$

$d^3 = 3.68 \times 10^6$ mm

$\Rightarrow$ d = 154.55 mm

$\therefore$ Taking the higher value of 'd'

We require a solid shaft of diameter D = 154.55 mm

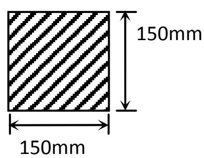
Q.6(a) What is the minimum actual length of the column for which Eular's formula hold good. If the cross section of uniform column is a square of side 150 mm. The column has one end hinged and other end fixed. **[10]**

Take, σ_c = 250 N/mm^2 & E = 200 GPa

Ans.: Given : Cross-section of given column = $150 \times 150 = 22500$ mm^2

Crushing stress (σ_c) = 250 N/mm^2

E = 200 GPa = 200×10^3 N/mm^2



Column end conditions,

One end is fixed and other is hinged

$\therefore$ Characteristics length is given as

$$L_e = \frac{L}{\sqrt{2}}$$

Here, crippling load by Euler formula,

$$P_{cr} = \frac{\pi^2 EI}{L_e^2}$$

$$P_{cr} = \sigma_c \times A = 250 \times 22500 \quad = 5625 \text{ kN}$$

$$\therefore \quad I = \frac{bd^3}{12}\ \frac{(150)(150^3)}{12} \quad = 42.1875 \times 10^6 \text{ mm}^4$$

$$(5625 \times 10^3) = \frac{(\pi^2)(200 \times 10^3)(42.1875 \times 10^6)}{L_e^2}$$

$\therefore$ Characteristic length, L_e = 3847.649 mm

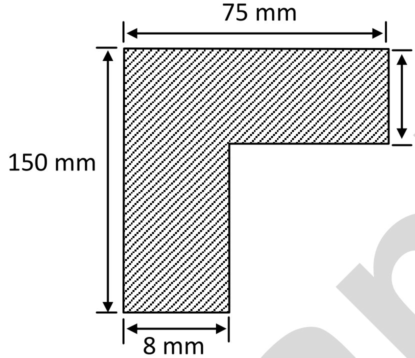
∴ Actual length of column for which Euler's formula hold good is

$L_e = \frac{L}{\sqrt{2}}$

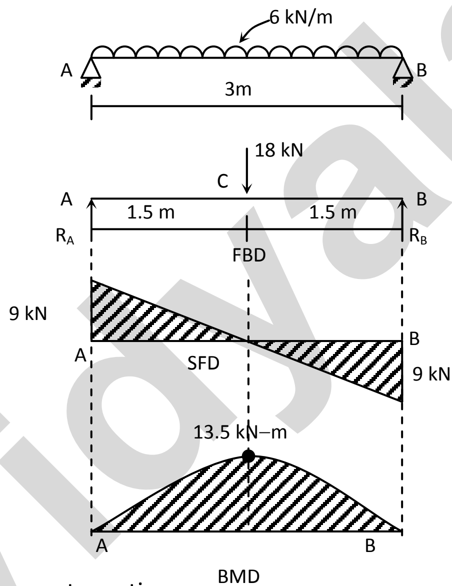
∴ $L = \sqrt{2} \, . \, L_e = \sqrt{2} \times 3847.649$

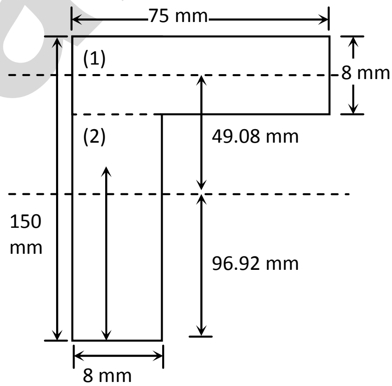
$= 5441.398$ mm

Q.6(b) An unequal angle as shown in fig. is used as a beam and carries a U.D.L. of 6 kN/m² over a span of 3 m find the maximum shear stress developmed and sketch the shear stress distriution across the section giving all important values. [10]



Ans.:





Support reaction,

$\sum f_y = 0$ ↑ +ve ↓ –ve

$R_A + R_B = 18$ kN

$\sum M_A = 0$ +), (–

$(18 \times 1.5) - R_B \times 3 = 0$

$R_B = 9$ kN, $R_A = 9$ kN

SF Calculation :

$SF|_{AL} = 0$ $SF|_{AR} = 9$ kN

$SF|_{CL} = 9$kN $SF|_{CR} = 9 - 18 = -9$ kN

$SF|_{BL} = -9$kN $SF|_{BR} = -9 + 9 = 0$ kN

BM Calculation :

$BM|_A = BM|_B = 14$ kN–m

$BM|_C = 9 \times 1.5 = 13.51$ kN–m

C.G. calculation (w.r.t to base)

Part	Area (A_i)	y_i	$A_i y_i$
Flange	$75 \times 8 = 600$	146	87600
Web	$142 \times 8 = 1136$	71	80656
	$\sum A_i = 1736$		$\sum A_i y_i = 168256$

$$= \frac{168256}{1736} = 96.92 \text{ mm}$$

M.I. Calculation:

$$I_{NA} = I_{NA}^{Flange} + I_{NA}^{Web} = \left[\left(\frac{75\times8^3}{12}\right) + (600)(49.08)^2\right] + \left[\left(\frac{8\times142^3}{12}\right) + (1136)(25.92)^2\right]$$

$$= 4.12 \times 10^6 \text{ mm}^4$$

Shear stress distribution for maximum stress :

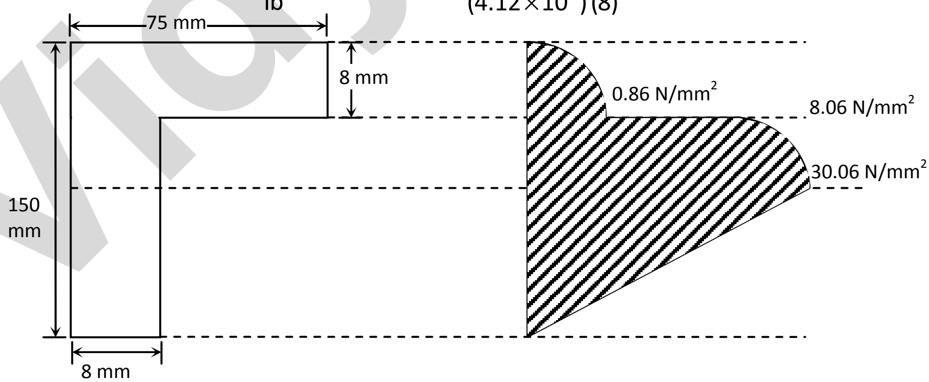
From SFD maximum shear force is 9 kN = 9×10^3 N

Shear stress at top edge = 0

Using shear stress, $\tau_{1\ flange} = \frac{FA\bar{y}}{Ib} = 0.85$ N/mm^2

$$\tau_{2\ (web)} = \tau_{1\ flange} \times \frac{\text{width of flange}}{\text{width of web}} = 0.86 \times \frac{75}{8} = 8.06 \text{ N/mm}^2$$

$$\tau_{NA} = \frac{F \times A_2 Y_2}{Ib} = \frac{(9\times10^3)(1136)(96.92)}{(4.12\times10^6)(8)} = 30.06 \text{ N/mm}^2$$



Cross Section of beam Shear Stress distribution

∴ Maximum shear stress in beam = 30.06 N/mm^2

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