

Q.1 Explain any FOUR of the following : **[20]**

Q.1(a) Prove that energy is a property of the system. **[5]**

Ans.: Consider a system which changes its state from state 1 to state 2 by following the path A, and returns from state 2 to state 1 by following the path B (Refer figure). So the system undergoes a cycle. Writing the first law for path A

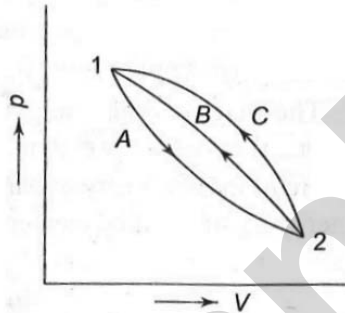


Fig.: Energy—A property of a system

$$Q_A = \Delta E_A + W_A \quad \dots(1)$$

and for path B

$$Q_B = \Delta E_B + W_B \quad \dots(2)$$

The processes A and B together constitute a cycle, for which

$$(\sum W)_{\text{cycle}} = (\sum Q)_{\text{cycle}}$$

$$\text{or } W_A + W_B = Q_A + Q_B$$

$$\text{or } Q_A - W_A = W_B - Q_B \quad \dots(3)$$

From Eqs. (1), (2), (3), it yields

$$\Delta E_A = -\Delta E_B \quad \dots(4)$$

Similarly, had the system returned from state 2 to state 1 by following the path C instead of path B

$$\Delta E_A = -\Delta E_C \quad \dots(5)$$

From Eqs. (4) and (5)

$$\Delta E_B = \Delta E_C \quad \dots(6)$$

Therefore, it is seen that the change in energy between two states of a system is the same, whatever path the system may follow in undergoing that change of state. If some arbitrary value of energy is assigned to state 2, the value of energy at state 1 is fixed independent of the path the system follows. Therefore, energy has a definite value for every state of the system. Hence, it is a point function and a property of the system.

The energy E is an extensive property. The specific energy, $e = E/M$ (J/kg), is an intensive property.

The cyclic integral of any property is zero, because the final state is identical with the initial state.

Q.1(b) Explain clausius inequality.

[5]

Ans.: Let us consider a cycle ABCD. Let Ab be a general process, either reversible or irreversible, while the other process in the cycle are reversible. Let the cycle be divided into number of elementary cycles.

For one of those elementary cycles, $\eta = 1 - \frac{dQ_2}{dQ}$.

Where dQ is the heat supplied at T and dQ₂ is the heat rejected at T₂

Now the efficiency of the general cycle will be less than or equal to the efficiency of the reversible cycle.

$$\eta_{irr} \geq \eta_{rev}$$

$$1 - \frac{dQ_2}{dQ} \leq \left(1 - \frac{dQ_2}{dQ} \right)_{Rev}$$

$$\text{or, } \frac{dQ_2}{dQ} \geq \left(\frac{dQ_2}{dQ} \right)_{Rev}$$

$$\text{or, } \frac{dQ}{dQ_2} \leq \left(\frac{dQ}{dQ_2} \right)_{Rav}$$

$$\text{But, } \left(\frac{dQ}{dQ_2} \right)_{Rav} = \frac{T}{T_2}$$

$$\text{Hence, } \frac{dQ}{dQ_2} \leq \frac{T}{T_2}$$

For a reversible process,

$$ds = \frac{dQ_{rev}}{T} = \frac{dQ_2}{T_2}$$

Hence for any process AB,

$$\frac{dQ}{T} \leq ds$$

$$\text{Then for cycle, } \oint \left(\frac{dQ}{T} \right) \leq \oint ds$$

Since entropy is a property and cyclic integral of any property is zero.

$$\oint \left(\frac{dQ}{T} \right) \leq 0$$

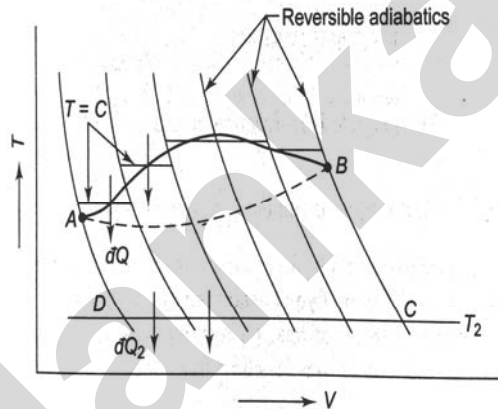
This equation is known as Clausius inequality.

It provides the criterion for the reversibility of the cycle.

$$\text{If, } \oint \left(\frac{dQ}{T} \right) = 0, \text{ the cycle is reversible.}$$

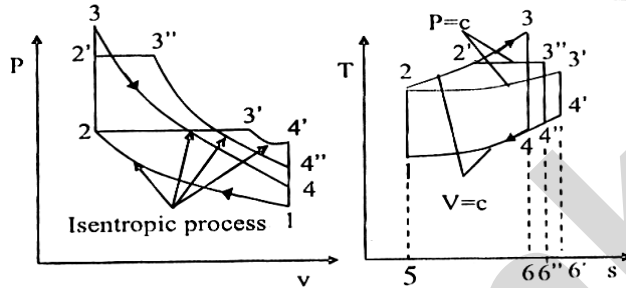
$$\oint \left(\frac{dQ}{T} \right) < 0, \text{ the cycle is possible and irreversible.}$$

$$\oint \left(\frac{dQ}{T} \right) > 0, \text{ the cycle is impossible.}$$



Q.1(c) For the same compression ratio and heat supplied compare Otto Diesel and Dual cycle with the help of P-V and T-S diagram. [5]
Dual cycle with the help of P-V and T-S diagram.

Ans.: From T-s diagram, it can be seen that Area 5236 = area 523'6' = area 522''6'' as this area represents the heat input which is the same for all cycles.
 It is seen from T-S diagram for the same heat input, the heat rejection in Otto cycle (Area 5146) is minimum and heat rejection in diesel cycle (Area 514'6') is maximum.
 Hence, Otto cycle has highest efficiency and Diesel has least. Dual cycle having the efficiency between the two

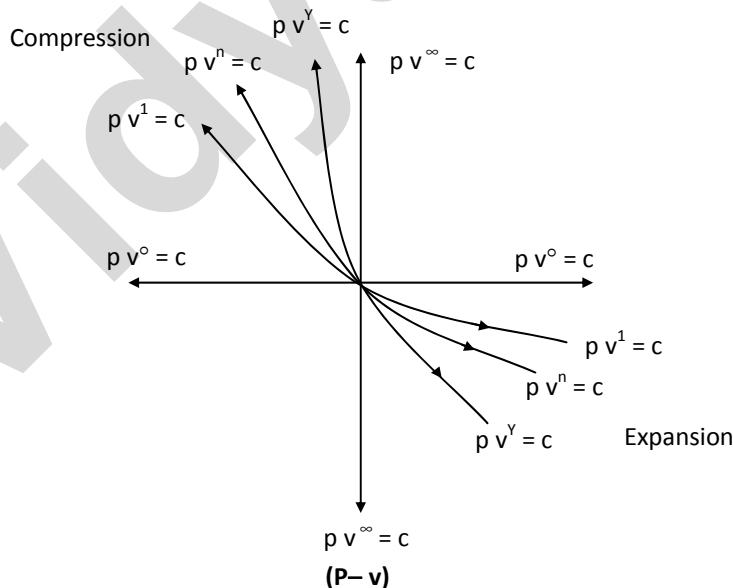


Q.1(d) Represent the following processes on PV and TS diagram starting from the same point. [5]

- (i) Isentropic process (ii) Isobaric process (iii) Isochoric process
- (iv) Isothermal process (v) Polytropic process.

Ans.: Representation of various thermodynamic processes on PV & T-S diagram. Depending on the value of (n) the index of expansion, the polytropic process reduces to other processes as if,

- $n = 0$: constant pressure process ($p = c$)
- $n = 1$: Constant temperature process ($T = C$)
- $n = \gamma$: reversible adiabatic process ($S = c$)
- $n = \infty$: constant volume process ($V = C$)



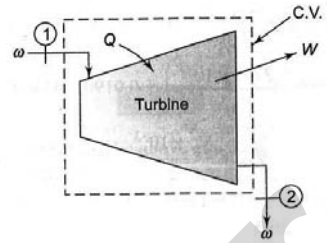
Ans.: Stead flow energy equation for the C.V. gives

$$w\left(h_1 + \frac{V_1^2}{2} + gZ_1\right) + \frac{dQ}{dt} = w\left(h_2 + \frac{V_2^2}{2} + gZ_2\right) + \frac{dW}{dt}$$

$$\therefore \frac{dW}{dt} = w\left[(h_1 - h_2) + \frac{V_1^2 - V_2^2}{2}\right] + \frac{dQ}{dt}$$

$$= 5\left[900 - 400 + \frac{50^2 - 150^2}{2} \times 10^{-3}\right] - 25 \times 5$$

$$= 2325 \text{ kW}$$



Using ideal gas equation of state a pipe inlet,

$$p_1 V_1 = w_1 R T_1$$

$$\dot{V} = \text{volume flow rate at inlet} = \frac{5 \times 0.285 \times 300 \times 10^3}{100 \times 10^3} = 4.275 \text{ m}^3/\text{s}$$

$$\text{Inlet area, } A_1 = \frac{4.275}{50} = 0.086 \text{ m}^2 = \frac{\pi}{4} D_1^2$$

$$\therefore D_1 = 0.33 \text{ m or } 33 \text{ cm}$$

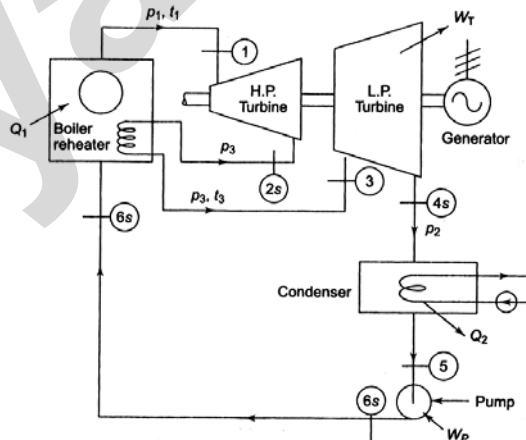
Q.2(b) Explain, how reheating and regeneration in Rankine cycle is beneficial. [6]

Ans.: Methods used to improve efficiency of Rankine cycle:

1. **Reheat cycle:** The reheat cycle is designed to take advantage of high boiler pressure by eliminating the problem of excessive moisture content in the exhaust steam.

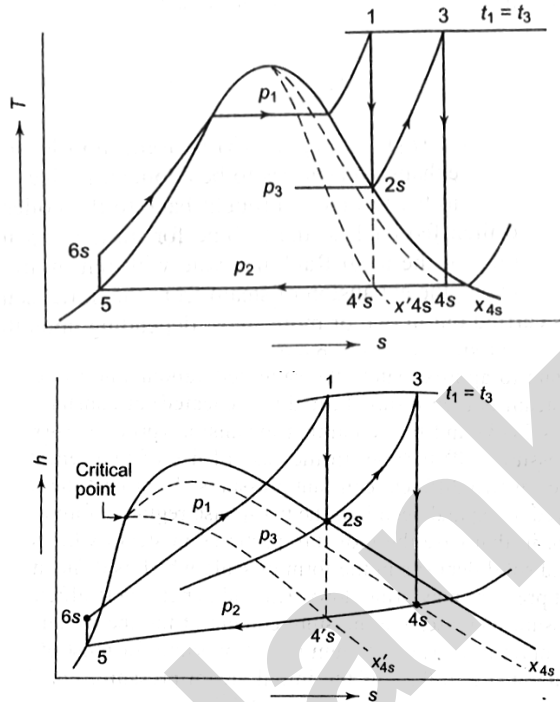
In the reheat cycle, the steam is expanded in number of stages.

After each stage of expansion, the steam is reheated in the boiler. Then it expands in next stage of turbine and is finally exhausted to condenser.



In the reheat cycle, the expansion of steam from initial state 1 to the condenser pressure is carried out in two or more stages, depending upon the number of reheats used. In the first step, steam expands in high pressure turbine from initial state to approximately the saturated vapor line (process 1-2s). The steam is then reheated at a constant pressure in boiler (process 2s-3) and the remaining expansion process (process 3-4s) is carried out in the low pressure turbine. In the

case of use of 2 re-heaters, steam is reheated twice at two different constant pressures.



Effects of reheat cycle:

- (a) Turbine work (W_T) increases.
- (b) Pump work (W_p) is constant.
- (c) Hence, net work done (W_{net}) increases.
- (d) Steam rate decreases.
- (e) Heat supplied increases.
- (f) Dryness fraction increases.
- (g) Condenser load increases.
- (h) Thermal efficiency may increase, decrease or constant.
- (i) If reheat pressure is too high the quality at exhaust will be less.
- (j) If reheat pressure is too low the thermal efficiency is less.

Hence we have to make compromise between these two.

Note: Practically in a reheat cycle the maximum number of reheats permitted is only two.

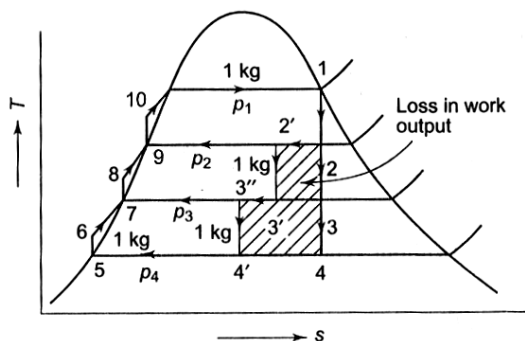
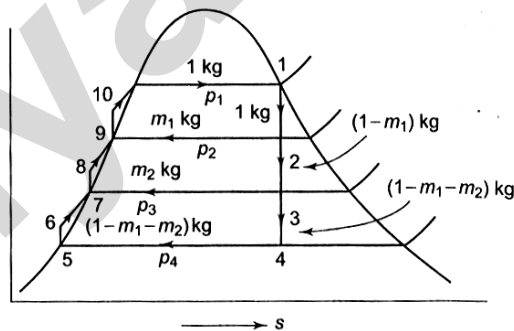
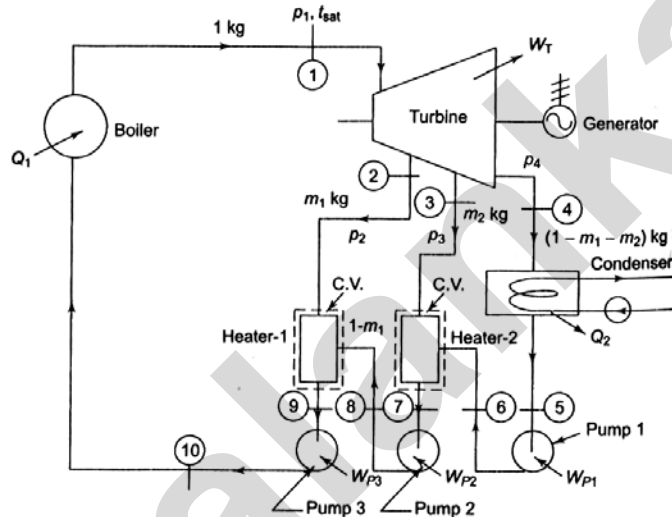
2. Regenerative cycle: The mean temperature of heat addition can also be increased by decreasing the amount of heat added at lower temperatures.

In a regenerative cycle, the feed water enters the boiler at a temperature between 4 and 4', and it is heated by steam extracted from intermediate stage of turbine.

For every kg of steam entering the turbine, let m_1 kg of steam be extracted from an intermediate stage of turbine. Where the pressure is p_2 and it is used to heat feed water. [(1-m) kg at state 8] by mixing in heater 1.

The remaining $(1-m)$ kg of steam then expands in the turbine from pressure p_2 (state 2) to pressure p_3 (state 3) when m_2 kg of steam is extracted for heating feed water in heater 2.

So $(1-m_1-m_2)$ kg of steam then expands in the remaining stages of turbine to pressure p_4 , gets condensed into water in condenser, and then pumped to heater 2, where it mixes with m_2 kg of steam extracted at pressure p_3 . Then $(1-m_1)$ kg of water is pumped to heater 1 where it mixes with m_1 kg of steam extracted at pressure p_2 . The resulting 1 kg of steam is then pumped to the boiler where heat from an external source is supplied.



Q.2(c) State the need of multistage reciprocating air compressor. [4]

Ans.: Advantages of Multistage compression :

1. The work done per kg of air is reduced in multistage compression with intercooler as compared to single stage compression for the same delivery pressure.
2. It improves the volumetric efficiency for the given pressure ratio.
3. The sizes of the two cylinders (i.e. high pressure and low pressure) may be adjusted to suit the volume and pressure of the air.
4. It reduces the leakage loss considerably.
5. It gives more uniform torque, and hence a smaller size flywheel is required.
6. It provides effective lubrication because of lower temperature range.
7. It reduces the cost of compressor.

Q.3(a) A reversible heat engine operates between two reservoirs at temperatures of 600°C and 40°C. The engine drives a reversible refrigerator which operates between reservoirs at temperatures of 40°C and –20°C. the heat transfer to the heat engine is 2000 KJ and the network output of the combined engine refrigerator plant is 360 KJ. Evaluate the heat transfer to the refrigerant and the net heat transfer to the reservoir at 40°C. [10]

Ans.: Maximum efficiency of the heat engine cycle (refer Figure) is given by

$$\eta_{\max} = 1 - \frac{T_2}{T_1} = 1 - \frac{313}{873} = 1 - 0.358 = 0.642$$

$$\text{Again } \frac{W_1}{Q_1} = 0.642$$

$$\therefore W_1 = 0.642 \times 2000 = 1284 \text{ kJ}$$

Maximum COP of the refrigerator cycle

$$(\text{COP})_{\max} = \frac{T_3}{T_2 - T_3} = \frac{256}{313 - 253} = 4.22$$

$$\text{Also } \text{COP} = \frac{Q_4}{W_2} = 4.22$$

$$\text{Since } W_1 - W_2 = W = 360 \text{ kJ}$$

$$\therefore W_2 = W_1 - W = 1284 - 360 = 924 \text{ kJ}$$

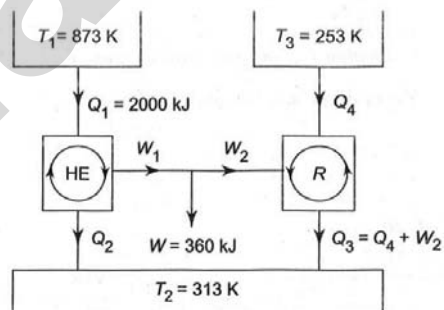
$$Q_4 = 4.22 \times 924 = 3899 \text{ kJ}$$

$$\therefore Q_3 = Q_4 + W_2 = 924 + 3899 = 4823 \text{ kJ}$$

$$Q_2 = Q_1 - W_1 = 2000 - 1284 = 716 \text{ kJ}$$

Heat rejection to the 40°C reservoir

$$= Q_2 + Q_3 = 716 + 4823 = 5539 \text{ kJ}$$



Q.3(b) Determine the maximum work obtainable from a Heat engine exchanging heat with two finite bodies of equal heat capacities at temperatures T_1 & T_2 ($T_1 > T_2$). [6]

Ans.: Maximum Work Obtainable from Two Finite Bodies at Temperature T_1 and T_2 .

Let us consider two identical finite bodies of constant heat capacity at temperatures T_1 and T_2 respectively, T_1 being higher than T_2 . If the two bodies are merely brought together into thermal contact, delivering no work, the final temperature T_f reached would be the maximum

$$T_f = \frac{T_1 + T_2}{2}$$

If a heat engine is operated between the two bodies acting as thermal energy reservoirs (Refer Figure), part of the heat withdrawn from body 1 is converted to work W by the heat engine, and the remainder is rejected to body 2. The lowest attainable final temperature T_f corresponds to the delivery of the largest possible amount of work, and is associated with a reversible process.

As work is delivered by the heat engine, the temperature of body 1 will be decreasing and that of body 2 will be increasing. When both the bodies attain the final temperature T_f , the heat engine will stop operating. Let the bodies remain at constant pressure and undergo no change of phase.

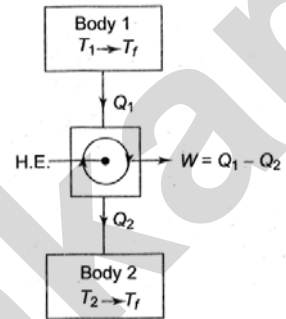


Fig.: Maximum work obtainable from two finite bodies

Total heat withdrawn from body 1

$$Q_1 = C_p(T_1 - T_f)$$

where C_p is the heat capacity of the two bodies at constant pressure.

Total heat rejected to body 2

$$Q_2 = C_p(T_f - T_2)$$

∴ amount of total work delivered by the heat engine

$$\begin{aligned} W &= Q_1 - Q_2 \\ &= C_p(T_1 + T_2 - 2T_f) \end{aligned} \quad \dots(1)$$

For given values of C_p , T_1 and T_2 , the magnitude of work W depends on T_f . work obtainable will be maximum when t_f is minimum.

Now, for body 1, entropy change ΔS_1 is given by

$$\Delta S_1 = \int_{T_1}^{T_f} C_p \frac{dT}{T} = C_p \ln \frac{T_f}{T_1}$$

For body 2, entropy change ΔS_2 would be

$$\Delta S_2 = \int_{T_2}^{T_f} C_p \frac{dT}{T} = C_p \ln \frac{T_f}{T_2}$$

Since the working fluid operating in the heat engine cycle does not undergo any entropy change, ΔS of the working fluid in heat engine $-\oint dS = 0$. Applying the entropy principle.

$$\Delta S_{\text{univ}} \geq 0$$

$$\therefore C_p \ln \frac{T_f}{T_1} + C_p \ln \frac{T_f}{T_2} \geq 0$$

$$C_p \ln \frac{T_f^2}{T_1 T_2} \geq 0 \quad \dots(2)$$

From eq. (2), for T_f to be a minimum

$$C_p \ln \frac{T_f^2}{T_1 \cdot T_2} = 0$$

$$\text{or} \quad \ln \frac{T_f^2}{T_1 T_2} = 0 = \ln 1$$

$$\therefore T_f = \sqrt{T_1 \cdot T_2} \quad \dots(3)$$

For W to be a maximum, t_f will be $\sqrt{T_1 T_2}$. from Eq. (1)

$$W_{\max} = C_p (T_1 + T_2 - 2\sqrt{T_1 T_2}) = C_p (\sqrt{T_1} - \sqrt{T_2})^2$$

The final temperatures of the two bodies, initially at T_1 and T_2 , can range from $(T_1 + T_2)/2$ with no delivery of work to $\sqrt{T_1 T_2}$ with maximum delivery of work.

Q.3(c) What is an irreversibility? State its types and causes. [4]

Ans.: Irreversibility :

- Reversible work, W_{rev} is the maximum amount of useful work that can be produced as a system undergoes a process between the specified initial and final states
- When the final state is dead state, the reversible work equals to energy.
- For a processes that require work, reversible work represents the minimum amount of work necessary to carry out that process.
- The difference between the reversible work. W_{rev} and the useful work, W is due to the irreversibilities.
- The irreversibility is equivalent to the exergy destroyed.
- For a totally reversible process. The actual and reversible work terms are identical, and thus the irreversibility is zero.
- Irreversibility represent the energy that could have been converted to work but was not because of the effects like frictional losses, heat transfer through finite temperature differences and so on.
- The irreversibility (I) of a process is defined as

$$I = W_r - W$$

Where, W_r = reversible work
 W = The actual work.

Causes of irreversibility :

The irreversibility of a process may be due to either one for both of the following :

1. Lack of equilibrium during the process
2. Involvement of dissipative effects

1. Irreversibility due to lack of equilibrium :

The lack of equilibrium (mechanical thermal or chemical) between the system and its surroundings , or between two systems, or two parts of the same system causes a spontaneous change which is irreversible. The following are specific example in this regards.

- (i) Heat transfer through a finite temperature difference
- (ii) Lack of pressure equilibrium within the interior of the system or between the system & the surrounding
- (iii) Free expansion.

2. Irreversibility due to dissipative effects :

The irreversibility of a process may be due to the dissipative effects in which work is done without producing an equivalent increase in the kinetic or potential energy of any system. The transformation of work into molecular internal energy either of the system or of the reservoir takes place through the energy of such phenomena as friction, viscosity, inelasticity, electrical resistance and magnetic hysteresis these effects are known as dissipative effects and work is said to be dissipated.

Types of irreversibility :
(1) Internal irreversibility :

The internal irreversibility is caused by the internal dissipative effects like/friction, turbulence, electrical resistance, magnetic hysteresis, etc, within the system.

(2) External irreversibility :

External irreversibility refers to the irreversibility occurring at the system boundary like heat interaction, with the surroundings due to a finite temperature gradient.

Q.4(a) Steam at 20 bar, 360 °C is expanded in a steam turbine to 0.08 bar. It then centers a condenser. Where it is condensed to saturated liquid water the pump feeds back the water into the boiler. [10]

Assuming ideal processes, find per kg of steam the network and the cycle efficiency.

Ans.: The property values at different state points (refer figure) found from the steam tables are given below :

$$\begin{aligned} h_1 &= 3159.3 \text{ kJ/kg} & s_1 &= 6.9917 \text{ kJ/kg K} \\ h_3 &= h_{f_2} = 173.88 \text{ kJ/kg} & s_3 &= s_{f_2} = 0.5926 \text{ kJ/kg K} \\ h_{f_{gp2}} &= 2403.1 \text{ kJ/kg} & s_{gp2} &= 8.2287 \text{ kJ/kg K} \\ v_{f_2} &= 0.001008 \text{ m}^3/\text{kg} & \therefore s_{f_{gp2}} &= 7.6361 \text{ kJ/kg K} \end{aligned}$$

$$\text{Now } s_1 = s_{2s} = 6.9917 = s_{f_2} + x_{2s} s_{f_{gp2}} = 0.5926 + x_{2s} \cdot 7.6361$$

$$\therefore x_{2s} = \frac{6.3991}{7.6361} = 0.838$$

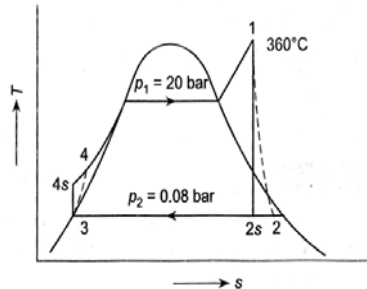
$$\therefore h_{2s} = h_{f_2} + x_{2s} h_{f_{gp2}} = 173.88 + 0.838 \times 2403.1 = 2187.68 \text{ kJ/kg}$$

$$\begin{aligned} \text{(a) } W_p &= h_{4s} - h_3 = v_{f_2} (p_1 - p_2) \\ &= 0.001008 \frac{\text{m}^3}{\text{kg}} \times 19.92 \times 100 \frac{\text{kN}}{\text{m}^2} \\ &= 2.008 \text{ kJ/kg} \end{aligned}$$

$$\begin{aligned} h_{4s} &= 175.89 \text{ kJ/kg} \\ W_T &= h_1 - h_{2s} \\ &= 3159.3 - 2187.68 = 971.62 \text{ kJ/kg} \end{aligned}$$

$$\begin{aligned} \therefore W_{\text{net}} &= W_T - W_p = 969.61 \text{ kJ/kg} \\ Q &= h_1 - h_{4s} = 3159.3 - 175.89 = 2983.41 \text{ kJ/kg} \end{aligned}$$

$$\therefore \eta_{\text{cycle}} = \frac{W_{\text{net}}}{Q_1} = \frac{969.61}{2983.41} = 0.325, \text{ or } 32.5\%$$



(b) If $\eta_p = 80\%$, and $\eta_T = 80\%$

$$W_p = \frac{2.008}{0.8} = 2.51 \text{ kJ/kg}$$

$$W_T = 0.8 \times 971.62 = 777.3 \text{ kJ/kg}$$

$$\therefore W_{\text{net}} = W_T - W_p = 774.8 \text{ kJ/kg}$$

$$\therefore \% \text{ reduction in work output} = \frac{969.61 - 774.8}{969.61} \times 100 = 20.1\%$$

$$h_{4s} = 173.88 + 2.51 = 176.39 \text{ kJ/kg}$$

$$Q_1 = 3159.3 - 176.39 = 2982.91 \text{ kJ/kg}$$

$$\therefore \eta_{\text{cycle}} = \frac{774.8}{2982.91} = 0.2597, \text{ or } 25.97\%$$

$$\therefore \% \text{ reduction in cycle efficiency} = \frac{0.325 - 0.2597}{0.325} \times 100 = 20.1\%$$

Q.4(b) State and derive steady flow energy equation and apply it to a turbine and nozzle. [5]

Ans.: Steady flow process: A steady flow process can be defined as a process during which all properties of fluid at each location within the system remain constant with respect to time i.e. the fluid properties can change from point to point within the control volume but at any fixed location they remain same during the entire process.

No properties like mass flow rate, energy flow rate, pressure, temperature, specific volume, specific internal energy etc. at an inlet or exit to the open system changes with time and remains constant.

Consider a steady flow device within a control volume as the working substance enters the device at the section 1 and leaves the section 2.

Let, Q = rate of heat transfer to control volume in W

W = rate of shaft work done by the fluid in control volume in W

p = pressure in pa

u = specific internal energy in J/kg

h = specific enthalpy in J/kg

V = velocity of fluid in m/s

z = elevation above datum in m

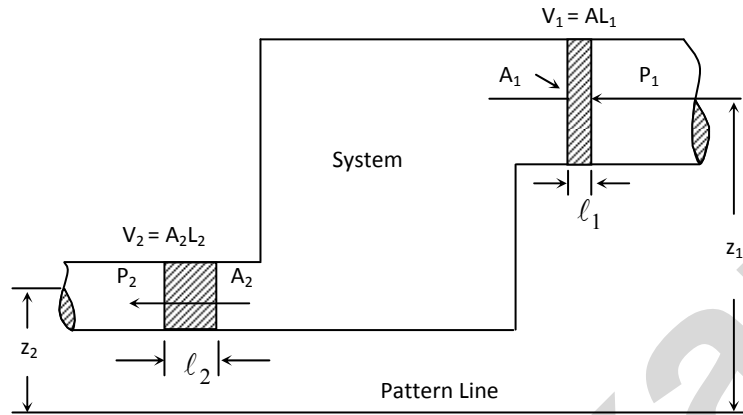
m_1 = mass flow rate into the control volume in kg/s

m_2 = mass flow rare out of the control volume in kg/s

By making mass balance equation between section 1 and 2,

$$m_1 = m_2$$

$$\rho_1 \cdot A_1 \cdot V_1 = \rho_2 \cdot A_2 \cdot V_2$$



By making energy balance equation between section 1 and 2,
Energy entering = Energy leaving

$$u_1 + p_1 \cdot V_1 + z_1 \cdot g + \frac{V_1^2}{2} + \frac{dQ}{dm} = u_2 + p_2 \cdot V_2 + z_2 \cdot g + \frac{V_2^2}{2} + \frac{dW}{dm}$$

$$h_1 + Z_1 \cdot g + \frac{V_1^2}{2} + \frac{dQ}{dm} = h_2 + Z_2 \cdot g + \frac{V_2^2}{2} + \frac{dW}{dm}$$

$$\frac{dQ}{dm} - \frac{dW}{dm} = (h_2 - h_1) + g(Z_2 - Z_1) + \frac{(V_2^2 - V_1^2)}{2}$$

The above equation is steady flow energy equation on mass basis, Multiplying by $\frac{dm}{dt}$ to convert SFEE on mass basis to SFEE on time basis.

$$\frac{dm}{dt} \left(h_1 + Z_1 \cdot g + \frac{V_1^2}{2} \right) + \frac{dQ}{dm} \frac{dm}{dt} = \frac{dm}{dt} \left(h_2 + Z_2 \cdot g + \frac{V_2^2}{2} \right) + \frac{dW}{dm} \frac{dm}{dt}$$

$$m \left(h_1 + Z_1 \cdot g + \frac{V_1^2}{2} \right) + Q = m \left(h_2 + Z_2 \cdot g + \frac{V_2^2}{2} \right) + W$$

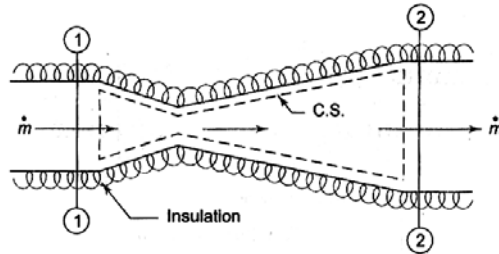
$$Q - W = (h_2 - h_1) + g(Z_2 - Z_1) + \left(\frac{V_2^2}{2} - \frac{V_1^2}{2} \right)$$

The SFEE applies to a wide variety of processes like flow through pipe line, heat transfer processes, mechanical power generation in engines, turbine, nozzles etc.

Application of SFEE:

(a) **Nozzle:** A nozzle is a device that increases the velocity of a fluid at the expense of pressure. The cross-sectional area of nozzle decreases with flow direction for sub-sonic flows and increases for super-sonic flow.

E.g. passages, propelling nozzles Steady flow energy equation:



$$Q - w = (h_2 - h_1) + g(Z_2 - Z_1) + \left(\frac{V_2^2}{2} - \frac{V_1^2}{2} \right)$$

But in this case of adiabatic flow, $Q = 0$ and $w = 0$.

$Z_2 = Z_1$ (no appreciable change in height)

$$(h_2 - h_1) + \left(\frac{V_2^2}{2} - \frac{V_1^2}{2} \right) = 0$$

When inlet velocity is very small compared to exit velocity,

$$(h_1 - h_2) = \frac{V_2^2}{2}$$

Therefore, $V_2 = \sqrt{2(h_1 - h_2)}$ m/s

So, as the velocity increases in the nozzle, the enthalpy decreases.

In the diffuser, however, as the velocity decreases, the enthalpy, and therefore temperature, rises.

(b) Turbine: Turbine converts heat energy into work. It gives positive work output.

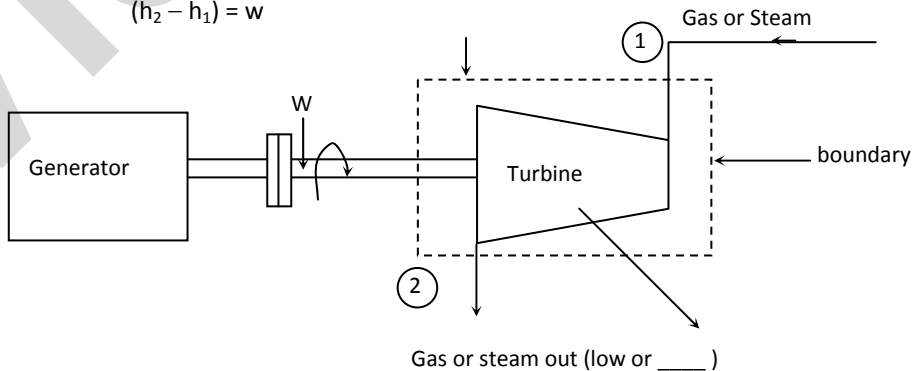
$$\text{SFEE, } Q - w = (h_2 - h_1) + g(Z_2 - Z_1) + \left(\frac{V_2^2}{2} - \frac{V_1^2}{2} \right)$$

For a turbine which is well insulated, SFEE becomes,

$$(h_2 - h_1) + \left(\frac{V_2^2}{2} - \frac{V_1^2}{2} \right) = w$$

Further, if change in kinetic energy is neglected, SFEE becomes,

$$(h_2 - h_1) = w$$



Q.4(c) State & explain Maxwell relations.

[5]

Ans.: A pure substance existing in single phase has only two independent variables. Of the 8 quantities p, V, T, S, U, H, F (Helmholtz function) and G (Gibbs function) anyone may be expressed as a function of any two others.

$$\text{If } dz = m \cdot dx + N \cdot dy \text{ then, } \left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y$$

For a pure substance undergoing an infinitesimal reversible process,

$$1. \quad dU = T \cdot dS - p \cdot dV$$

Since U is thermodynamic property of exact differentials,

$$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial p}{\partial S}\right)_V$$

$$2. \quad dH = dU + p \cdot dV + V \cdot dp = T \cdot dS + V \cdot dp$$

Since H is thermodynamic property of exact differentials,

$$\left(\frac{\partial T}{\partial p}\right) = \left(\frac{\partial V}{\partial S}\right)_p$$

$$3. \quad dF = dU - T \cdot dS - S \cdot dT = -P \cdot dV - S \cdot dT$$

Since F is thermodynamic property of exact differentials,

$$\left(\frac{\partial p}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T$$

$$4. \quad dG = dH - T \cdot dS - S \cdot dT = V \cdot dp - S \cdot dT$$

Since G is thermodynamic property of exact differentials,

$$\left(\frac{\partial V}{\partial T}\right)_p = \left(\frac{\partial S}{\partial p}\right)_T$$

Q.5(a) A system contains 0.15 m^3 of air at 3.8 bar and 150°C . reversible adiabatic expansion takes place till pressure falls to 1.03 bar . The gas is then heated at constant pressure till enthalpy increases by 60.7 KJ . Determine total work done. If these processes are replaced by a single reversible polytrophic process giving same work between initial and final states. Determine index of expansion. [10]

Ans.: $T_1 = 150^\circ\text{C} = 273 + 150 = 473 \text{ k}$

mass flow rate of air (m)

Using ideal gas equation

$$p_1 v_1 = mRT_1$$

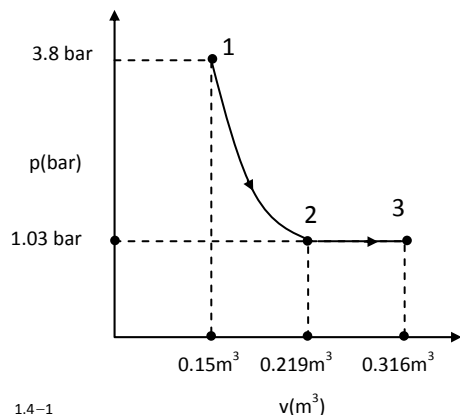
$$300 \times 0.15 = m \times 0.287 \times 473$$

$$\therefore m = 0.469 \text{ kg}$$

• Process (1–2) : Adiabatic expansion :

$$\therefore \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{T_2}{T_1}\right)$$

$$\therefore T_2 = T_1 \times \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = 473 \times \left(\frac{1.03}{3.8}\right)^{\frac{1.4-1}{1.4}} = 291.31 \text{ k}$$



$$\text{Also, } \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$$\left(\frac{1.03}{3.8}\right)^{\frac{1.4-1}{1.4}} = \left(\frac{0.15}{V_2}\right)^{1.4-1}$$

$$\therefore V_2 = 0.219 \text{ gm}^3$$

Process (2 – 3) : Constant pressure ($p = c$)

Enthalpy (H) of air increased by 60.7 kJ

$$\therefore H = m.C_p.(T_3 - T_2)$$

$$\therefore 60.7 = 0.469 \times 1.005 \times (T_3 - 291.31)$$

$$\therefore T_3 = 420 \text{ k}$$

$$\therefore \frac{V_2}{T_2} = \frac{V_3}{T_3} \Rightarrow V_3 = \frac{T_3}{T_2} \times V_2 = \frac{420}{291.31} \times 0.219 = 0.316 \text{ m}^3$$

\therefore work done in process 1–2 (W_{1-2})

$$W_{1-2} = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1} = \frac{(380 \times 0.15) - (103 \times 0.219)}{1.4 - 1} = 86.108 \text{ kJ}$$

Work done in process 2–3 : (W_{2-3})

$$W_{2-3} = Pdv = P(v_3 - v_2) = 103 (0.316 - 0.219) = 10 \text{ kJ}$$

Total work done (W) = (W_{1-2}) + (W_{2-3})

$$= 86.108 + 10$$

$$W = 96.108 \text{ kJ}$$

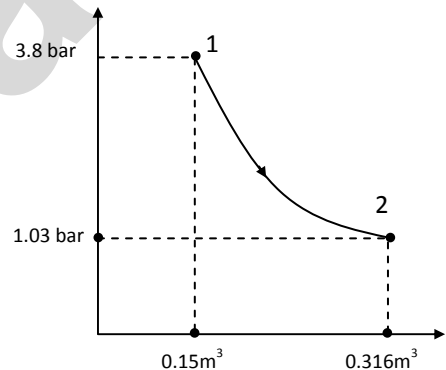
Now, the above process is replaced by single polytropic process from initial to final state work done in polytropic process is given by

$$W = \frac{P_1 V_1 - P_2 V_2}{n - 1}$$

$$\therefore 96.108 = \frac{(380 \times 0.15) - (103 \times 0.316)}{n - 1}$$

$$\therefore n = 1.254$$

index of polytropic compression (n) = 1.254



Q.5(b) Write the statement of second law of thermodynamic and establish the equivalence between them. [5]

Ans.: Second law of thermodynamics.

Efficiency of the engine is given by,

$$\eta = \frac{W_{\text{net}}}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

We know that for any engine, $W_{\text{net}} < Q_1$, since heat Q_1 transferred to a system cannot be completely converted to work in a cycle. Therefore efficiency is always less than unity. Hence, $Q_2 > 0$ i.e. there has always to be a heat rejection. To produce network in a thermodynamic cycle, a heat engine has thus to exchange heat with two reservoir, the source and the sink.

Kelvin-Plank statement of second law of thermodynamics: It is impossible for a heat engine to produce network in complete cycle if it exchanges heat only with bodies at a single fixed temperature. Heat flows from a high temperature body to low temperature body. The reverse process can never occur spontaneously.

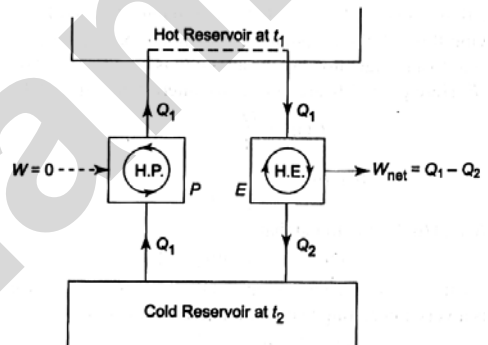
Clausius statement of second law of thermodynamics: It is impossible to construct a device which, operating in a cycle, will produce no effect other than the transfer of heat from a low temperature body to high temperature body.

Establish an equivalence between Kelvin-Plank statement and Clausius statement of second law of thermodynamics.

It appears that Kelvin-Plank statement and the Clausius statement of second law of thermodynamics are altogether different, but they are equivalent.

The equivalence of the two statements will be proven if it can be shown that the violation of one statement implies the violation of second and vice versa.

- Let us consider a cyclic heat pump (P) which transfers heat from low temperature reservoir (at t_1) to a high temperature reservoir (at t_2) with no other effect i.e. with e no expenditure of work which violates Clausius statement.

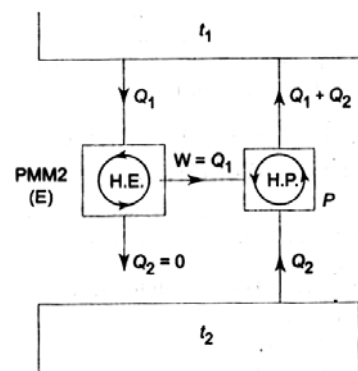


Let us assume a cyclic heat engine (E) operating between the same reservoirs, producing work (W_{net}) in a single cycle. The engine is operated such that it draws heat (Q_1) from the hot reservoir equal to heat discharged by the heat pump. Then the hot reservoir may be eliminated and heat (Q_1) discharged by the heat pump is fed to the heat engine.

So we see that the heat pump (P) and the heat engine (E) acting together constitutes a heat engine operating in a cycle and producing network while exchanging heat only with one body at a single fixed temperature (t_2). This violates the Kelvin-Plank statement.

- Let us now consider a heat engine (E) produces network in a cycle by exchanging heat with only one reservoir (at t_1) i.e. PMM-2 which violates Kelvin-Plank statement.

Let us assume a cyclic heat pump (P) extracting heat (Q_2) from a low temperature reservoir (at t_2) and discharging heat to the high temperature reservoir (at t_1) with the expenditure of work (W) equal to work produced by heat engine (E).



So we see that the heat engine (E) and the heat pump (P) acting together constitutes a heat pump operating in a cycle and producing the sole effect of transfer of heat from a low temperature to high temperature body without any external work.

This violates the Clausius statement.

Q.5(c) Derive the expression for cycle efficiency of otto cycle. [5]

Ans.: Otto cycle: The main drawback of the Carnot cycle is its impracticality due to high pressure and high volume ratios employed with comparatively low mean effective pressure.

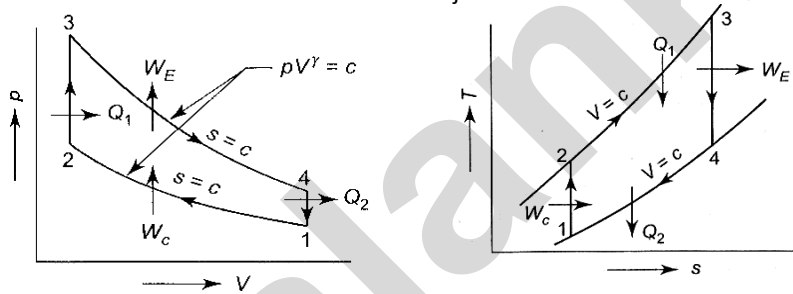
Nicolas Otto (1876) thus proposed a constant volume heat addition cycle which forms a basis for working of today's spark ignition engines.

Process 1-2: Reversible adiabatic compression of air when piston moves upward.

Process 2-3: Reversible constant volume heat addition

Process 3-4: Reversible adiabatic expansion.

Process 4-1: Reversible constant volume heat rejection.



Air standard efficiency:

Compression ratio: It is defined as the ratio of volume at the beginning of the compression to the volume at the end of the compression.

$$r_k = \frac{\text{Volume at the beginning of the compression}}{\text{Volume at the end of compression}} = \frac{V_1}{V_2}$$

The thermal efficiency of Otto cycle can be written as,

$$\eta_{\text{Otto}} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

Let m be the fixed mass of air undergoing the cycle,

Heat supplied, $Q_{(2-3)} = Q_1 = m \cdot C_v \cdot (T_3 - T_2)$

Heat rejected, $Q_{(4-1)} = Q_2 = m \cdot C_v \cdot (T_4 - T_1)$

Hence, efficiency can be given as,

$$\eta_{\text{Otto}} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{m \cdot C_v \cdot (T_4 - T_1)}{m \cdot C_v \cdot (T_3 - T_2)} = 1 - \frac{T_4 - T_1}{T_3 - T_2} \quad \dots (1)$$

For process 1-2, $\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$

or, $T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1}$

For process 3-4, $\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$

or, $T_3 = T_4 \left(\frac{V_1}{V_2}\right)^{\gamma-1}$

Putting the value of T_2 and T_3 in equation (1),

$$\eta_{\text{Otto}} = 1 - \frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{T_4 - T_1}{T_4 \left(\frac{V_1}{V_2}\right)^{\gamma-1} - T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1}} = 1 - \frac{1}{r_k^{(\gamma-1)}}$$

- Q.6(a)** A single stage reciprocating air compressor has a swept volume of 2000 cm³ [10] and runs at 800 r.p.m. It operates on a pressure ratio of 8, with a clearance of 5% of the swept volume. Assume NTP room conditions and at inlet ($P = 101.3$ KPa, $t = 15^\circ\text{C}$), and polytropic compression and expansion with $n = 1.25$. Calculate : (i) Indicated power (ii) Volumetric efficiency (iii) mass flow rate (iv) FAD (v) Isothermal efficiency and (vi) The actual power needed to drive the compressor, if mechanical efficiency is 0.85.

Ans.: $p_1 = 101.3$ kPa,
 $p_2 = 8p_1 = 810.4$ kPa
 $T_1 = 288$ K,
 $V_s = 2000$ cm³
 $V_3 = V_c = 0.05 V_s = 100$ cm³
 $V_1 = V_c + V_s = 2100$ cm³
 $p_3 V_3^n = p_4 V_4^n$

$$\therefore V_4 = \left(\frac{p_3}{p_4}\right)^{\frac{1}{n}} \cdot V_3 = (8)^{\frac{1}{1.25}} \times 100 = 528 \text{ cm}^3$$

$$V_1 - V_4 = 2100 - 528 = 1572 \text{ cm}^3$$

$$W = \frac{n}{n-1} p_1 (V_1 - V_4) \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1 \right]$$

$$= \frac{1.25}{0.25} \times 101.3 \times 10^3 \times 1572 \times 10^{-6} \left[(8)^{\frac{0.25}{1.25}} - 1 \right] = 411 \text{ J}$$

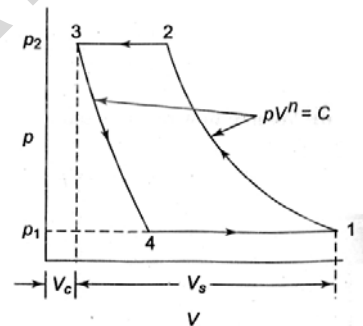
(a) Indicated power = $\frac{411 \times 800 \times 10^{-3}}{60} = 5.47 \text{ kW}$

(b) Volumetric efficiency = $\frac{1572}{2000} \times 100 = 78.6 \%$

(c) Mass of air compressed per cycle

$$m = \frac{pV}{RT} = \frac{101.3 \times 10^3 \times 1572 \times 10^{-6}}{287 \times 288} = 1.93 \times 10^{-3} \text{ kg}$$

$$\therefore \text{Mass flow rate} = 1.93 \times 10^{-3} \times 800 = 1.54 \text{ kg/min}$$



(d) FAD = Free air delivery = $1572 \times 10^{-6} \times 800 = 1.26 \text{ m}^3/\text{min}$

(e) $W_t = p_1 (V_1 - V_4) \ln \frac{p_2}{p_1} = 101.3 \times 10^3 \times 1572 \times 10^{-6} \ln 8 = 331 \text{ J}$

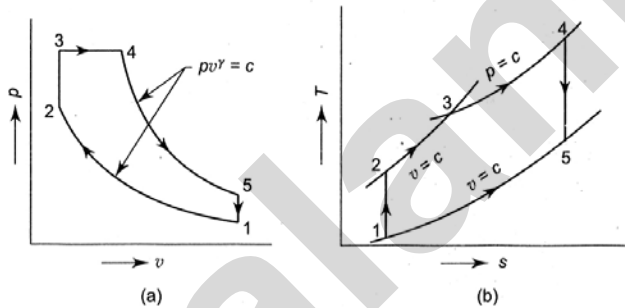
$$\eta_{\text{isothermal}} = \frac{0.331 \times 800}{5.47 \times 60} = 80.7 \%$$

(f) Input power = $\frac{5.47}{0.85} = 6.44 \text{ kW}$

Q.6(b) An air standard dual cycle has a compression ratio of 16 and compression begins at 1 bar 50°C. The maximum pressure is to bar. The heat transferred to air at constant pressure is equal to that at constant volume. Estimate :

- (i) The pressures and temperatures at the cardinal points of the cycle,
- (ii) The cycle efficiency, and
- (iii) The m.e.p. of the cycle, $cr = 0.718 \text{ KJ/kg K}$

Ans.: Given (refer Figure)



$$T_1 = 273 + 50 = 323 \text{ K}$$

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2} \right)^{\gamma-1} = (16)^{0.4}$$

$$\therefore T_2 = 979 \text{ K}$$

$$p_2 = p_1 \left(\frac{v_1}{v_2} \right)^{\gamma} = 1.0 \times (16)^{1.4} = 48.5 \text{ bar}$$

$$T_3 = T_2 \cdot \frac{p_3}{p_2} = 979 \times \frac{70}{48.5} = 1413 \text{ K}$$

$$Q_{2-3} = c_v (T_3 - T_2) = 0.718(1413 - 979) = 312 \text{ kJ/kg}$$

Now $Q_{2-3} = Q_{3-4} = c_p (T_4 - T_3)$

$$\therefore T_4 = \frac{312}{1.005} + 1413 = 1723 \text{ K}$$

$$\frac{v_4}{v_3} = \frac{T_4}{T_3} = \frac{1723}{1413} = 1.22$$

$$\therefore \frac{v_5}{v_4} = \frac{v_1}{v_2} \times \frac{v_3}{v_4} = \frac{16}{1.22} = 13.1$$

$$\therefore T_5 = T_4 \left(\frac{v_4}{v_5} \right)^{\gamma-1} = 1723 \times \frac{1}{(13.1)^{0.4}} = 615 \text{ K}$$

$$p_5 = p_1 \left(\frac{T_5}{T_1} \right) = 1.0 \times \frac{615}{323} = 1.9 \text{ bar}$$

$$\begin{aligned} \eta_{\text{cycle}} &= 1 - \frac{Q_2}{Q_1} = 1 - \frac{c_v (T_5 - T_1)}{c_v (T_3 - T_2) + c_p (T_4 - T_3)} \\ &= 1 - \frac{0.718 (615 - 323)}{312 + 312} = 1 - \frac{0.718 \times 292}{624} = 0.665 \text{ or } 66.5 \% \end{aligned}$$

$$v_1 = \frac{RT_1}{p_1} = \frac{0.287 \text{ kJ/kgK} \times 323 \text{ K}}{10^2 \text{ kN/m}^2} = 0.927 \text{ m}^3/\text{kg}$$

$$v_1 - v_2 = v_1 - \frac{v_1}{16} = \frac{15}{16} v_1$$

$$W_{\text{net}} = Q_1 \times \eta_{\text{cycle}} = 0.665 \times 624 \text{ kJ/kg}$$

$$\begin{aligned} \therefore \text{m.e.p.} &= \frac{W_{\text{net}}}{v_1 - v_2} = \frac{0.665 \times 624 \text{ kJ/kg}}{\frac{15}{16} \times 0.927 \text{ m}^2/\text{kg}} \\ &= 476 \text{ kN/m}^2 \\ &= 4.76 \text{ bar} \end{aligned}$$

