Prelim Paper **Applied Mathematics - III**

Time: 3 Hrs.] **[Marks : 80**

N.B.: (1) Question No. **1** is **compulsory**.
(2) Attempt any **THREE** of the remaining.

1. (a) Find Laplace transform of $te^{3t}\cos t$. **[5]**

(b) Evaluate $\oint_c \frac{z-1}{z^2+2z+5}dz$, where c is $|z+1+i| = 2$. **[5]**

(c) Show that f(z) = sinh z is analytic. Hence find its derivative. **[5]**

(d) Compute spearman's rank correlation for the data : **[5]**

X :	18	20	34	52	12
Y :	39	23	35	18	46

2. (a) Show that the function $w = \frac{4}{z}$ transforms the straight line x = c in the z-plane into circle is w-plane. Find its centre and radius. **[6]**

(b) Show that $\int_0^\infty e^{-t}\int_0^t \frac{\sin u}{u}du\,dt = \frac{\pi}{4}$ **[6]**

(c) Obtain fourier series for $f(x) = \begin{cases} 1+\frac{2x}{\pi} & -\pi \le x \le 0 \\ 1-\frac{2x}{\pi} & 0 \le x \le \pi \end{cases}$ **[8]**

Hence deduce that $\frac{\pi^2}{8} = \frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots$

3. (a) Evaluate $\oint_c \frac{e^{kz}}{z}dz$, where c : $|z| = 1$. Hence deduce that $\int_0^\pi e^{k\sin\theta}\cos(k\sin\theta)\,d\theta = \pi$ **[6]**

(b) For the lines of regression $6y - 5x = 90$, $15x - 8y = 130$ and $\sigma_x^2 = 16$ **[6]**
Find (i) $\bar{x}, \bar{y}$ (ii) r (iii) σ_y

(c) Solve the differential equation $\frac{dy}{dx}+2y+\int_0^t y\,dt = \sin t$ using Laplace transform give y(0) = 1 **[8]**

4. (a) Find Laurentz's series for $f(z) = \frac{2}{(z-1)(z-2)}$ indicating region of convergence. **[6]**

(b) Show that $\cos x = \frac{8}{\pi}\sum_{m=1}^{\infty}\frac{m}{4m^2-1}\sin(2mx)$ If $0 < x < \pi$ **[6]**

(c) Find Bilinear transformation which maps the points 1, i, −1 onto the points i, 0, −i. Hence find fixed points and image of $|z| < 1$. **[8]**

5. (a) Solve using Bender-Schmidt method $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$, subject to the conditions **[6]**

$u(0, t) = 0$, $u(1, t) = 0$, $u(x, 0) = \sin \pi x$, $0 \le x \le 1$

(b) Determine the solution of one-dimensional heat equation under the boundary conditions, $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ **[6]**

$u(0, t) = 0$, $u(\ell, t) = 0$, $u(x, 0) = x$, $(0 < x < \ell)$, ℓ being length of the rod.

(c) Find inverse Laplace Transform of **[8]**

(i) $\log\left(1 + \frac{\alpha^2}{s^2}\right)$ (ii) $\frac{e^{-s}}{s^2 + s + 1}$

6. (a) Obtain complex form of fourier series for $F(x) = e^{\alpha x}$, in $(-\pi, \pi)$ where a is not an integer. **[6]**

(b) Fit a curve $y = a \cdot b^x$ to the following data, using method of least squares. **[6]**

X :	2	3	4	5	6
Y :	144	172.8	207.4	248.8	298.5

(c) (i) Evaluate $\int_0^{2\pi} \frac{d\theta}{5 + 3\sin\theta}$ using Residue Theorem. **[4]**

(ii) Evaluate $\int_{-\infty}^{\infty} \frac{dx}{x^2 + 1}$ using Residue Theorem. **[4]**

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