Q.1 Attempt the following (any FOUR) : 20

Q.1(a) What is pitot tube? On what principle does it works? 05

Ans.: Pitot Tube

It is a device used for measuring the velocity of flow at any point in a pipe or a channel. It is based on the principle that if the velocity of flow at a point becomes zero. The pressure there is increased due to the conversion of the kinetic energy into pressure energy. In its simplest form, the pitot tube consist of a glass tube bent at right angles as shown in fig. the lower end which is bent through 90° is directed in the upstream direction the liquid rises up in the tube due to the conversion of kinetic energy into pressure energy. The velocity is determined by measuring the rise of liquid in the tube.

Consider two points (1) and (2) at same level in such a way that point (2) is just as the inlet of the pitot tube and point (1) is far away from the tube.

P_1 P_2 = intensity of pressure at point (1) and (2) respectively

V_1V_2 = velocity at point (1) and (2) respectively

H = depth of tube in the liquid

h = rise of liquid in the tube above the free surface.

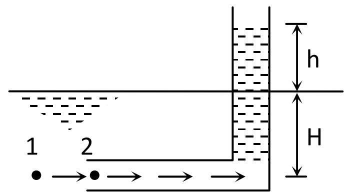


Fig.: Pitot tube

Applying Bernoulli equation at point (1) and (2), we get

$$\frac{P_1}{\rho g} + \frac{V_1^2}{\rho g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{\rho g} + Z_2$$

But $Z_1 = Z_2$ as points (1) and (2) are on the same line and $V_2 = 0$.

$$\frac{P_1}{\rho g} = \text{pressure head} = (H) \qquad \ldots \text{at (1)}$$

$$\frac{P_2}{\rho g} = \text{Pressure head} = (h + H) \qquad \ldots \text{at (2)}$$

Substituting this values, we get

$$H + \frac{V_1^2}{2g} = (h + H)$$

$$\therefore \quad h = \frac{V_1^2}{2g}$$

or $\quad V_1 = \sqrt{2g\,h}$

This is theoretical velocity. Actual velocity is given by

$$(V_1)_{actual} = C_v \sqrt{2gh}$$

Where, C_V = Coefficient of pitot tube.

Q.1(b) What is boundary layer development and factors that affect growth of boundary layer? 05

Ans.: Boundary Layer Development :

- Boundary layer is that thin layer of fluid in the immediate vicinity of bounding surface (Solid body over which velocity gradients and shear stresses are large.
- The boundary layer starts at the leading edge of the solid boundary and the boundary layer grows thickness increases with the distance 'x' along the surface.
- The most important changes takes place in the nature of flow as the boundary layer grows this is known as development as boundary layer.
- The boundary layer begin at the leading edge of the plate, where the flow is laminar and velocity profile is approximately parabolic.
- Due to the increase in the thickness of boundary layer the laminar layer the becomes unstable and the motion within it becomes disturbed.
- The irregularities of flow developed into turbulence and the thickness of the layer increases more rapidly.
- The short length over which the nature of flow changes from laminar to turbulent is called as transition zone.
- The velocity in turbulence zone is logarithmic.

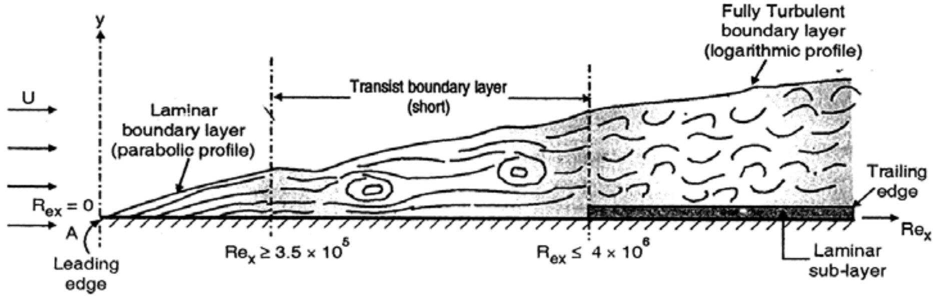


Fig.: Boundary layer on long, flat surface with sharp leading edge.

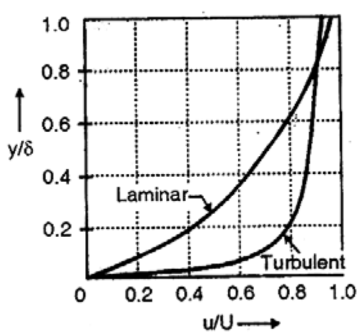


Fig.: Typical velocity profiles in laminar and turbulent boundary layer.

Factors Affecting growth of boundary layer :

1) **Distance 'X' from the leading-edge:** It varies directly with distance 'x', more the distance 'x' the thickness of boundary layer is also more.

2) **Free stream velocity :** Boundary layer varies inversely as the free stream velocity 'U' if free stream velocity increases, thickness of boundary layer decreases and vice-versa.
3) **Viscosity of fluid :** Boundary layer varies directly with viscosity if viscosity of fluid is more and vice–versa
4) **Density of fluid :** Boundary layer varies inversely with density for lower density fluid thickness is more and vice-versa.

Q.1(c) Write short note on drag and lift. **05**

Ans.: Drag and Lift :

- Consider a body held stationary in a stream of real fluid moving at uniform velocity U.
- The force acting at any point on the small element dA of the surface of the body are resolve into two component's i.e. shear force τ.dA and pressure force P.dA acting along the tangential direction and Normal direction to the surface.
- The sum of components of the shear forces in the direction of flow of fluid is called as frictional drag. (F_{DF}) or skin drag or shear drag.

  $\therefore$ Friction drag, $F_{DF} = \int_A \tau \cdot dA \cdot \cos\theta$
- Similarly the sum of components of the pressure forces in the direction of the fluid motion is caused as pressure drag (F_{DP}) or form drag.

  $\therefore$ Pressure drag, $F_{DP} = \int_A P \cdot dA \cdot \sin\theta$

  $\therefore$ Total drag, $F_D = F_{DF} + F_{DP}$
- The lift on the body is given by the summation of the components of shear and pressure forces acting over the body in the direction perpendicular to the direction of fluid motion.

  $$F_L = \int_A \tau \cdot dA \sin\theta + \int_A P \cdot dA \cos\theta$$
- Equation of drag and lift is given as, $F_D = C_D \times \frac{1}{2}\rho U^2 \cdot A$

  $F_L = C_L \times \frac{1}{2}\rho U^2 \cdot A$

  where, C_D = Coefficient of drag
  C_L = Coefficient of lift

Q.1(d) Explain : (i) Local acceleration (Temporal acceleration) **[5]**
(ii) Convective acceleration

Ans.: (i) Local acceleration / Temporal acceleration :

- The rate of increase of velocity with respect to time at a given point in a flow field is called as local acceleration.
- For steady flow local acceleration is zero,

  $$a = \left(\frac{\partial u}{\partial t}\right)$$

(ii) Convective acceleration :

- The rate of increase of velocity with respect to changes in position of fluid particle in a flow field is called as convective acceleration for steady flow, convective acceleration is not zero in uniform flow the convective acceleration is zero.

$$a = \left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right)$$

Q.1(e) What are the significance of "Mach Number"? On the basis of match number define : (i) Sonic flow, (ii) Subsonic flow, (iii) supersonic flow. **05**

Ans.: **Mach Number :**

- Ratio of characteristic velocity 'v' to the velocity of sound waves 'c' is known as mach number

$$M = \sqrt{\frac{\text{Inertiq force}}{\text{Elastic force}}} = \sqrt{\frac{\rho A V^2}{YA}} = \frac{V}{C}$$

- It can also be expressed as square root of the ratio of inertial force per unit volume $\left(\frac{\rho V^2}{\forall}\right)$ to the elastic force per unit volume $\left(\frac{E}{\forall}\right)$ ($\forall$= volume)

- **Significance :**

– Mach number is important in those problems in which the characteristic velocity is comparable with the velocity of sound or velocity of elastic wave.
– The cases related to aero planes bullets and projectiles deals with Mach number.
– Mach number is also very important in the case of flow of gases through orifices, nozzles and venturimeters.
– On the basis of Mach number flows can be classified into following catagories.
  (i) **Sonic flow :** The flow for which Mach number is equal to unity (M = 1).
  (ii) **Subsonic flow :** For this Mach number 'M' is in between range 0.4 to 1 ($M \leq 1$).
  (iii) **Supersonic flow :** Mach number for this flow is always greater than unity but less than six ($1 < M < 6$).

Q.2(a) A Fluid flow field is given by $\bar{u} = xy\,i + 2yj - (yz + 2z)k$ **10**

Determine whether this is a possible steady incompressible fluid flow. If so, calculate magnitude of acceleration at the point (1, 2, 3).

Ans.: Given flow equation :

$$\bar{u} = xyi + 2yj - (yz + 2z)\,k$$

to determine whether the flow is in compressible, steady and possible. Continuity equation must satisfy.

Continuity Equation :

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \quad \text{... (i)}$$

Here, $u = xy$ $\quad v = 2y$ $\quad w = -(yz + 2z)$

$$\frac{\partial u}{\partial x} = y, \quad \frac{\partial v}{\partial y} = 2, \quad \frac{\partial w}{\partial z} = -(y + 2)$$

Putting in (i) we get,

$y + 2 - (y + z) = 0$

Hence, flow is possible in compressible and steady.

To determine acceleration of flow, we have

$$a_x = \frac{dy}{dt} = u\frac{du}{dx} + v\frac{du}{dx} + w\frac{du}{dz} + \frac{du}{dt}$$

$$= u(y) + vx + w(0) + 0 \quad = uy + vx$$

$$a_x = (xy^2 + 2xy) \qquad \text{... (ii)}$$

$$a_y = \frac{dv}{dt} = u\frac{dv}{dx} + v\frac{dv}{dy} + w\frac{dv}{dz} + \frac{dv}{dt}$$

$$= u(0) + v(2) + w(0) + 0$$

$$a_y = 2v = 2(2y) \quad = 4y \qquad \text{... (iii)}$$

$$a_z = \frac{dw}{dt} = u\frac{dw}{dx} + v\frac{dw}{dy} + w\frac{dw}{dz} + \frac{dw}{dt}$$

$$= u(0) + v(-z) + w[-(y + 2)] + 0$$

$$= -vz + wy - 2w$$

$$= 2yz - [-(yz + 2z)\, y] - 2\, [-(yz + 2z)]$$

$$= -2yz + (y^2z + 2yz) + (2yz + 4z)$$

$$= -2yz + y^2z + 2yz + 2yz + 4z$$

$$a_z = y^2z + 2yz + 4z \qquad \text{... (iv)}$$

We have resultant acceleration of flow (a_{res})

$$a_{res} = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

a_x, a_y, a_z at (1, 2, 3) are as follows :

$a_x = (xy^2 + 2xy) \quad = 1(2)^2 + 2(1)(2) \quad = 4 + 4 \quad = 8$

$a_y = (4y) \quad = 4(2) \quad = 8$

$a_z = (y^2z + 2yz + 4z) \quad = 2(2)(3) + 2^2(3) + 4(3) \quad = 12 + 12 + 12 \quad = 36$

$a_{res} = \sqrt{8^2 + 8^2 + 36^2} \quad = \sqrt{1424} \quad = 37.735$

Q.2(b) How Bernoulli's equation is based on Euler's theory of fluid flow and is obtained by integrating the Euler's equation of motion? **10**

Ans.: Euler's equation of motion :

This is equation of motion in which the forces due to gravity and pressure are taken into consideration this is derived by considering the motion of fluid element along a stream line.

Consider a stream line in which flow is taking place in direction as shown in figure. Consider a cylindrical element of cross-section dA and length ds the forces action on the cylindrical element are :

1) Pressure force pdA in direction of flow
2) pressure force $\left(p + \frac{\partial p}{\partial s}ds\right) d\,\Delta$ opposite to direction of flow.
3) weight of element ρgd Ads.

Let 'θ' is the angle between the direction of flow and the line of action of the weight of elements.

The resultant force on fluid element indirection of 's' must be equal to the mass of the fluid element × acceleration in direction 's'.

$$pdA - \left(p + \frac{\partial p}{\partial s}ds\right)dA - \rho gdAds\cos\theta = p.dA.ds \times a_s \qquad \text{... (i)}$$

Where a_s is acceleration in the direction of 's'

$$a_s = \frac{dv}{dt} \qquad \text{where v is a function of s and t}$$

$$= \frac{\partial v}{\partial s}\frac{ds}{dt} + \frac{\partial v}{\partial t} \qquad = \frac{v\partial v}{\partial s} + \frac{\partial V}{\partial t} \qquad \left\{\frac{ds}{dt} = v\right\}$$

If the flow is steady, $\frac{\partial v}{\partial t} = 0$

$$a_s = \frac{v\partial v}{\partial s}$$

Substituting the value of as in eq. (i) and simplifying the equation

$$-\frac{\partial p}{\partial s}dsdA - \rho gdA\,ds\cos\theta = pdAds \times \frac{\partial v}{\partial s}$$

Dividing by pdρdA,

$$-\frac{\partial P}{\rho\partial s} - g\cos\theta = \frac{v\partial v}{\partial s}$$

Or $\frac{\partial p}{\rho\partial s} + g\cos\theta + v\frac{\partial v}{\partial s} = 0$

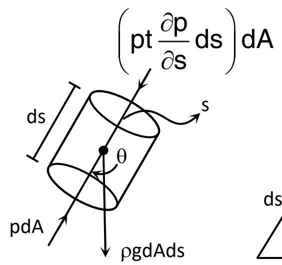


Fig. (a)

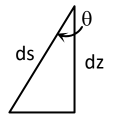


Fig. (b)

From figure (b) we have

$$\cos\theta = \frac{dz}{ds}$$

$$\frac{1}{p}\frac{dp}{ds} + g\frac{dz}{ds} + \frac{vdv}{ds} = 0 \qquad \text{or} \qquad \frac{dp}{\rho} + g\,dz + v\,dv = 0$$

$$\frac{dp}{s} + g\,dz + v\,dv = 0 \qquad \text{... (A)}$$

This is known as Euler's equation of motion.

Bernoull's equation is obtained by integrating the Euler's equation of motion (A) as,

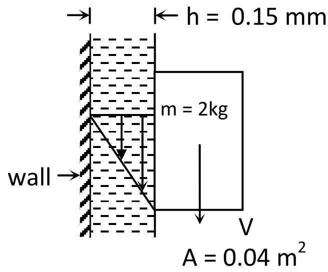
$$\int\frac{dP}{\rho} + \int gdz + \int vdv = \text{constant}$$

If flow is incompressible, 'ρ' is constant and

$$\therefore \quad \frac{P}{\rho} + gz + \frac{V^2}{2} = \text{constant} \qquad \text{Or} \quad \frac{P}{\rho g} + \frac{V^2}{2g} + z = \text{constant}$$

Where, $\frac{p}{\rho g}$ = pressure head, $\frac{V^2}{2g}$ = kinetic head, Z = potential head

Q.3(a) **A solid block 2 kg mass slides steadily at a velocity 'V' along a vertical wall as shown in fig. below. A thin oil film of thickness h = 0.15 mm provides lubrication between the block and wall. The surface area of the face of the block in contact with the oil film is 0.04 m^2. The velocity distribution within the oil gap is liner as shown in figure. Take dynamic viscosity of oil as 7 × 10^{-3} pa–s and acceleration due to gravity as 10 m/s^2. Neglect weight of the oil the terminal velocity 'V' (m/s) of the block is?** **[10]**



Ans.: Terminal velocity is a constant velocity i.e. the net acceleration is zero.

So, $\Sigma F_{net} = ma$

$$mg - \tau A = 0$$

$$\tau A = mg$$

We know, $\tau = \mu \dfrac{du}{dy}$

$$\mu \frac{u}{h} \cdot A = mg$$

$$7 \times 10^{-3} \times \frac{u}{0.15 \times 10^{-3}} \times 0.04 = 2 \times 10$$

$$u = 10.714 \text{ m/s}$$

Q.3(b) **Define following terms:** **10**

(i) streamline (ii) streak line (iii) path line (iv) source (v) sink

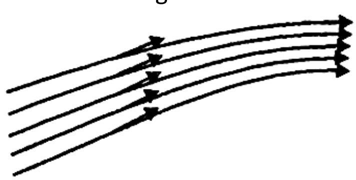
Ans.: **(i) Streamline :**

- The imaginary line drawn in the fluid in such a way that the tangent to any point gives the direction of motion at that point; is called as stream line.
- In steady flow, the pattern of streamline invariant with time.
- Suppose a particle is moving along the stream line from A to B through a distance ds in a small interval of time dt.
- Let dx, dy, dz be the component of ds along x, y, z axis respectively if u be the velocity of fluid particle, then the time taken by particle to move from A to B is dt,

$$dt = \frac{ds}{u} \qquad \text{where, u = velocity in x direction}$$

Thus equation of a streamline in a three dimensional flow is given as

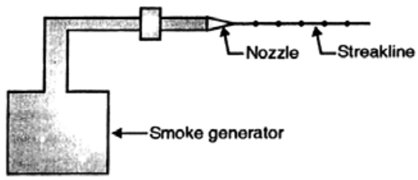
$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$



(ii) Streate line :

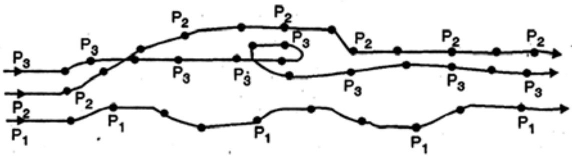
- The curve that gives an instantaneous picture of the location of the fluid particles, which have passed through a given point, is called as streak line.

- For example, the line formed by smoke particles ejected from nozzle or smoke coming out from chimney. A colour dye when injected into the flowing fluid and a resultant coloured filament lines at a given location gives streak lines.



(iii) Pathline :

- The Actual path followed by fluid particle as it moves during a period of time is called as pathline.
- The path line shows the direction of the velocity of same particles at successive instant of time.
- A path line can intersect itself.



(iv) Source :

- If the two dimensional motion of an ideal fluid consists of an out word radial flow from a point and is symmetrical in all directions then the point is call a simple source.

(v) Sink :

- For a fluid flows, a sink is a negative source and is a point of in word radial flow at which the fluid is considered to be absorbed or annihilated.

Q.4(a) What is stream function and velocity potential function? Explain relation between both. How equipotential line and streamline are perpendicular to each other? **10**

Ans.: Stream function :

It is defined as the scalar function of space and time. Such that its partial derivative with respect to any direction gives the velocity component at right angles to that direction. It is de noted by ψ (psi) and defined only two dimensional flows.

For steady flow, $\psi = f(x, y)$

such that, $\frac{\partial \psi}{\partial x} = v \qquad \frac{\partial \psi}{\partial y} = -u$

Velocity Potential Function :

It is defined as a scalar function of space and time such that its negative derivative with respect to any direction gives the fluid velocity in that direction. It is denoted by $\phi = F(x, y, z)$

$$u = -\frac{\partial \phi}{\partial x}, \quad v = -\frac{\partial \phi}{\partial y}, \quad w = -\frac{\partial \phi}{\partial z}$$

Relation between stream function & velocity potential function :

Potential function $u = -\frac{\partial \phi}{\partial x}$, $v = -\frac{\partial \phi}{\partial y}$

Stream function $u = -\frac{\partial \psi}{\partial y}$, $v = -\frac{\partial \psi}{\partial x}$

$$u = -\frac{\partial \phi}{\partial x} = -\frac{\partial \psi}{\partial y} \qquad v = -\frac{\partial \phi}{\partial y} = \frac{\partial \psi}{\partial x}$$

$$\therefore \quad \frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}$$

$$\Rightarrow \quad \therefore \quad \frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x}$$

Equipotential line and streamline are perpendicular :

Equipotential line is the line along which velocity potential function (ϕ) is constant.

We know, ϕ = constant for equipotential line and

$\phi = f(x, y)$ for steady flow

$$d\phi = \frac{\partial \phi}{\partial x} \cdot dx + \frac{\partial \phi}{\partial y} dy \qquad \text{... (i)}$$

By definition $\frac{\partial \phi}{\partial x} = -u$, $\frac{\partial \phi}{\partial y} = -v$ and $\partial \phi = 0$

Substituting in eq. (i),

$0 = -u\,dx - v\,dy$

$u dx = -v dy$

$-\frac{u}{v} = \frac{dy}{dx}$ is the slope of equipotential line

Slope $m_1 = \frac{dy}{dx} = -\frac{u}{v}$

Consider a streamline is the line alone which stream function ψ is constant and

$\psi = F(x, y)$

$$d\psi = \frac{\partial \psi}{\partial x} dx + \frac{\partial \psi}{\partial y} dy$$

by definition $\frac{\partial \psi}{\partial x} = V$, $\frac{\partial \psi}{\partial y} = -u$ and $\partial \psi = 0$.

$0 = v\,dx - u\,dy$

$\frac{v}{u} = \frac{dy}{dx}$ is he slope of a streamline.

$\therefore$ Slope $m_2 = \frac{dy}{dx} = \frac{v}{u}$

Condition of perpendicular lines

$$m_1 \cdot m_2 = -\frac{u}{v} \times \frac{v}{u} = -1$$

∴ Equipotential line and streamline are orthogonal/perpendicular to each other.

Q.4(b) Three pipes of 400 mm, 200 mm and 300 mm diameters have lengths of 400m, 200m and 300m respectively. They are connected in series to make a compound pipe the ends of this compound pipe are connected with two tanks whose difference of water level is 16m. If co-efficient of friction for these pipe is same & equal to 0.005. Determine the discharge through the compound pipe neglecting the minor losses. **10**

Ans.: Here, Three pipes are connected in series,

Given : Difference of water levels, H = 16 m

Length and diameter of pipe 1, L_1 = 400 m and d_1 = 400 mm = 0.4 m

Length and diameter of pipe 2, L_2 = 200 m and d_2 = 200 mm = 0.2 m

Length and diameter of pipe 3, L_3 = 300 m and d_3 = 300 mm = 0.3 m

Also, $f_1 = f_2 = f_3 = 0.005$

Discharge through the compound pipe first neglecting minor losses.

Let v_1, v_2 and v_3 are the velocities in the 1st, 2nd and 3rd pipe respectively.

From continuity, we have $A_1V_1 = A_2V_2 = A_3V_3$

$$v_2 = \frac{A_1 v_1}{A_2} = \frac{\frac{\pi}{4} d_1^2}{\frac{\pi}{4} d_2^2} \times v_1 = \left(\frac{0.4}{0.2}\right)^2 v_1 = 4v_1$$

$$v_3 = \frac{A_1 v_1}{A_3} = \frac{\frac{\pi}{4} d_1^2}{\frac{\pi}{4} d_3^2} \times v_1 = \frac{d_1^2}{d_3^2} v_1 = \left(\frac{0.4}{0.2}\right)^2 v_1 = 1.77\ v_1$$

Now, according to Darcy's equation, we have

$$H = \frac{4F_1L_1V_1^2}{d_1 \times 2g} + \frac{4F_2L_2V_2^2}{d_2 \times 2g} + \frac{4F_3L_3V_3^2}{d_3 \times 2g}$$

$$16 = \frac{4 \times 0.005 \times 400 \times V_1^2}{0.4 \times 2 \times 9.81} + \frac{4 \times 0.005 \times 200 \times (4V_1)^2}{0.2 \times 2 \times 9.81} + \frac{4 \times 0.005 \times 300}{0.3 \times 2 \times 9.81}(1.77\ v_1)^2$$

$$= \frac{V_1^2}{2 \times 9.81}\left(\frac{4 \times 0.005 \times 400}{0.4} + \frac{4 \times 0.005 \times 200 \times 16}{0.2} + \frac{4 \times 0.005 \times 300 \times 3.157}{0.3}\right)$$

$$16 = \frac{V_1^2}{2 \times 9.81}(20 + 320 + 63.14)$$

$$16 = \frac{V_1^2}{2 \times 9.81} \times 403.14$$

$$V_1 = \sqrt{\frac{16 \times 2 \times 9.81}{403.14}} = 0.882 \text{ m/s}$$

∴ Discharge, $Q = A_1 \times V_1 = \frac{\pi}{4} \times (0.4)^2 \times 0.882 = 0.1108 \text{ m}^2/\text{s}$

Q.5(a) Find shape factor for the velocity distribution in the boundary layer given by **10**

$$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$$

Where, u is the velocity at the distance 'y' from the surface of the flat plate and 'U' be the free stream velocity at the boundary layer thickness 'δ'.

Ans.: Given velocity distribution,

$$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$$

(i) Displacement thickness δ^* is given by

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy$$

Substituting the value of $\frac{u}{U} = 2\left(\frac{u}{\delta}\right) - \left(\frac{u}{\delta}\right)^2$, we have

$$\delta^* = \int_0^\delta \left\{1 - \left[2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2\right]\right\} dy \qquad = \int_0^\delta \left\{1 - 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2\right\} dy$$

$$= \left[y - \frac{2y^2}{2\delta} + \frac{y^3}{3\delta^2}\right]_0^\delta \qquad = \delta - \frac{\delta^2}{\delta} + \frac{\delta^3}{3\delta^2} \qquad = \delta - \delta + \frac{\delta}{3}$$

$$= \frac{\delta}{3} \qquad \dots \text{(i)}$$

(ii) Momentum thickness θ, is given by,

$$\theta = \int_0^\delta \frac{u}{v}\left\{1 - \frac{u}{U}\right\} dy \quad = \int_0^\delta \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right) \left[1 - \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)\right] dy$$

$$= \int_0^\delta \left[\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right] \left[1 - \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)\right] dy \quad = \int_0^\delta \left[\frac{2y}{\delta} - \frac{4y^2}{\delta^2} + \frac{2y^3}{\delta^3} - \frac{y^2}{\delta^2} + \frac{2y^3}{\delta^3} - \frac{y^4}{\delta^4}\right] dy$$

$$= \int_0^\delta \left[\frac{2y}{\delta} - \frac{5y^2}{\delta^2} + \frac{4y^3}{\delta^3} - \frac{y^4}{\delta^4}\right] dy \qquad = \left[\frac{2y^2}{2\delta} - \frac{5y^3}{3\delta^2} + \frac{4y^4}{4\delta^3} - \frac{y^5}{5\delta^4}\right]_0^\delta$$

$$= \left[\frac{\delta^2}{\delta} - \frac{5\delta^3}{3\delta^2} + \frac{\delta^4}{\delta^3} - \frac{\delta^5}{5\delta^4}\right] \qquad = \delta - \frac{5\delta}{3} + \delta - \frac{\delta}{5}$$

$$= \frac{15\delta - 25\delta + 15\delta - 3\,\delta}{15} \qquad = \frac{30\delta - 28\delta}{15}$$

$$= \frac{2\delta}{15} \qquad \dots \text{(ii)}$$

Shape factor is a ratio of displacement thickness to momentum thickness.

$$\therefore \quad \text{Shape factor} = \frac{\delta^*}{\theta} \quad = \frac{\delta/3}{2\delta/15} \quad = \frac{\delta}{3} \times \frac{15}{2\delta} \quad = \frac{5}{2}$$

$$\text{Shape factor} = \frac{5}{2}$$

Q.5(b) An orifice meter with orifice diameter 15 cm is inserted in a pipe of 30 cm diameter. The pressure difference measured by a mercury oil differential manometer on the two sides of the orifice meter gives a reading of 50 cm of mercury. Find the rate of flow of oil of specific gravity 0.9 when the coefficient of discharge of the orifice meter = 0.64. **10**

Ans.: Given : Diameter of orifice, $d_0 = 15$ cm

$\therefore$ Area, $a_0 = \frac{\pi}{4}(15)^2 = 176.7\ \text{cm}^2$

Diameter of pipe, $d_1 = 30$ cm

$\therefore$ Area, $a_1 = \frac{\pi}{4}(30)^2 = 706.85\ \text{cm}^2$

Specific gravity of oil, $S_0 = 0.9$

Reading of differential manometer, x = 50 cm of mercury.

$\therefore$ Differential head $h = x\left[\frac{s_g}{s_0} - 1\right] = 50\left[\frac{13.6}{0.9} - 1\right]$ cm of oil

$= 50 \times 14.11 = 705.5$ cm of oil

$C_d = 0.64$ (given)

$\therefore$ The rate of the flow, Q is given by equation.

$$Q = C_d \frac{a_0 a_1}{\sqrt{a_1^2 - a_0^2}} \times \sqrt{2gh}$$

$$= 0.64 \times \frac{176.7 \times 706.85}{\sqrt{(706.85)^2 - (176.7)^2}} \times \sqrt{2 \times 9.81 \times 705.5} = \frac{94046317.78}{684.4}$$

$= 137414.25\ \text{cm}^3/\text{s} = 137.414$ litre/s

Q.6(a) Explain following terms: **10**

(i) Circulation and vorticity **(ii) Total pressure and centre of pressure**

(iii) Streamlined body and bluff body **(iv) Lagrangian and Eulerian method**

Ans.: **(i) Circulation and vorticity :**

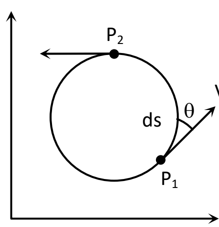
- **Circulation :** It is the line integral of the tangential component of the velocity taken around a closed contour.

– ρ treamline forms closed loop, shown in figure the velocity of streamline at any point (p_1, p_2) is tangential to the radius of curvature 'R' rotating at an angular velocity 'w'

– Consider ds-small length of streamline

– Circulation (γ) $= \oint V_t \cdot ds$

$$\gamma = \int_0^{2\pi} \omega R \cdot R d\theta = 2\pi\omega R^2$$



- **Vorticity (Ω) :** Vorticity is a ratio of circulation to area

$$\therefore \quad \Omega = \frac{2\pi\,\omega R^2}{\pi R^2} = 2\omega$$

$\therefore \quad \Omega = 2\omega$

Hence, at any point vorticity is equal to two times α rotation (ω).

(ii) Total pressure and centre of pressure :

- Total pressure: Force exerted by a static fluid on a surface either plane or curved when the fluid comes in contact with the surface this force always acts normal to the surface.

$$F = \rho g A \bar{h}$$

- Centre of pressure (h*): It is the point of application of the total pressure on the surface. It is calculated by using "Principle of moments'

$$h^* = \frac{I_G + Ah^2}{A\bar{h}}$$

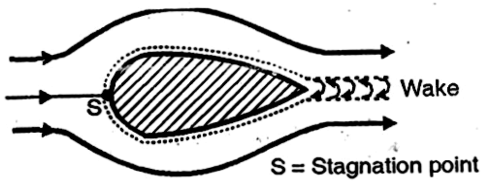
(iii) Streamlined body and bluff body :

- **Streamlined body :** The body whose surface coincides with the streamlines when the body is placed in a flow is called as stream line body.

– The flow become turbulent from laminar but does not separate upto rear most edge of the body. Thus the wake formation zone at rear most edge will be very small.

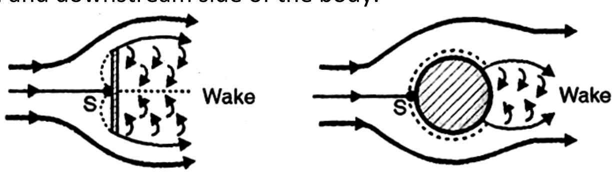
– Consequently the pressure drag will be very small and hence the total drag will be due to friction only.

– Submarine, torpedo, spaceship, aero plane are the examples of the streamlined body.



- **Bluff body:** The body whose surface does not coincide with the streamlines when placed in a flow is called as bluff body.

– A very wide wake is developed on the downstream of the body due to which pressure on downstream falls considerably and produces the pressure difference on the upstream and downstream side of the body.



a) Circular disc b) Sphere or cylinder

(iv) Lagrangian and Eulerian method :

- **Lagrangian method :**

– In this method the observer concentrates on the movement of single particle.

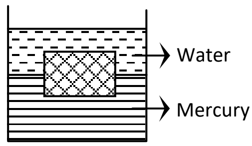
– Observer has to move with the particle movement.

– The path followed by the particle and changes in its velocity acceleration, density etc. are described. In this method, the observer moves with motion of fluid.
– This method is complex and it's become difficult to understand fluid motion.

- **Eulerian method :**

– In this method observer concentrates on the fixed point particles.
– Observer remains stationary and observes changes in the fluid parameters at the particular point only.
– This method is simple and is very commonly adapted to understand the fluid motion.

Q.6(b) Find the density of a metallic body which floats at the interface of mercury of specific gravity 13.6 and water such that 40% of its volume is submerged in mercury and 60% in water. **10**



Ans.: Let the volume of the body = Vm^3

Then volume of body sub-merged in mercury $= \frac{40}{100} V = 0.4\ Vm^3$

Volume of body sub-merged in water $= \frac{60}{100} \times V = 0.6\ Vm^3$

For the equilibrium of the body.

Total buoyant force (upward force) = weight of the body

But total buoyant force = force of buoyancy due to water + Force of buoyancy due to mercury

(1) Force of buoyancy due to water = weight of water displaced by body
= Density of water × g × volume or water displaced
= 1000 × g × volume of body in water
= 1000 × g × 0.6 × V Newton

(2) Force of buoyancy due to mercury = weight of mercury displaced by body
= g × Density of mercury × volume of mercury displaced
= [g × 13.6 × 1000 × 0.4 V] Newton

Weight of the body = Density × g × volume of body = ρgV

Where ρ – is density of the body

∴ For equilibrium, we have

Total buoyant force = weight of the body

$(1000 \times g \times 0.6 \times V) + (13.6 \times 1000 \times g \times 0.4\ v) = \rho gV$

$\rho = 600 + (13600 \times 0.4) = 600 + 54400$

$= 6040.00\ Kg/m^3$

∴ Density of the body = $6040.00\ Kg/m^3$

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